LONGMANS'

JUNIOR SCHOOL MENSURATION

TO MEET THE REQUIREMENTS OF THE
OXFORD AND CAMBRIDGE JUNIOR LOCAL EXAMINATIONS
THE COLLEGE OF PRECEPTORS, ETC.

BY

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PREFACE.

THE practical value of Arithmetic is its application; this book therefore may be regarded as a continuation of the course laid down in the JUNIOR SCHOOL ARITHMETIC.

Only those portions of Mensuration which can be mastered by pupils who have a good knowledge of vulgar fractions, decimals and square root are dealt with in the following questions, and unnecessarily long and tedious arithmetical calculations have been avoided.

Simple demonstrations of the methods employed are given, that young scholars may have an intelligent knowledge of the principles on which the rules of Mensuration are based; numerous diagrams illustrate the various figures and solids.

Great care has been taken to graduate the questions; every rule is dealt with in easy stages, and one difficulty only is presented at a time. Short mechanical exercises for accuracy are in every section followed by problems of a practical nature.

Questions set at the University Local Examinations, the Examinations of the College of Preceptors and the Education Department have been adopted in all parts of the book.

The JUNIOR SCHOOL MENSURATION not only fulfils the requirements of students preparing for the above examinations, but also covers the syllabus of Continuation Evening Schools and Technical Classes.

THE MODERN SCHOOL, FAREHAM, HANTS,
December 1892.

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Part I.

MENSURATION OF SURFACES.

CHAPTER I.

DEFINITIONS AND UNITS OF MEASUREMENTS.

1. **Mensuration** is that branch of mathematics by which we find the length, area, and volume of a body.

2. Every body is said to have three **dimensions**, namely, length, breadth, and thickness (or depth).

3. The boundary or the limit of a body from the rest of space is called its **surface** or **superficies**; thus we speak of the surface of a lake without any regard to its depth.

4. A surface which is perfectly flat and even is called a **plane surface**; for instance, the top of a table. There are surfaces which are not flat or even, such as the curved surface of a globe.

5. The extent of a surface, that is, the space included within its boundaries, is called its **area**; thus the area of a field is the quantity of land contained in it.

6. The measurement of the length of lines and the area of surfaces is called **mensuration of surfaces.**

7. The boundaries or limits of a surface are **lines**; thus we speak of the edge or line limiting the surface of a table without any regard to its extent.

8. When lines intersect or cross each other, the place where they intersect is called a **point**.

It is evident that a surface has only two dimensions, length and breadth; a line only one, length; whilst a point has no dimensions, but marks position only.

9. The shortest distance between two points is called a **straight line.**

10. Straight lines are said to be **parallel** when they are everywhere the same distance from each other, and if carried out ever so far, on either side, will never meet; for instance, the lines of a railway.

11. When two straight lines, having different directions, meet together at a point, the opening between them is called an **angle**.

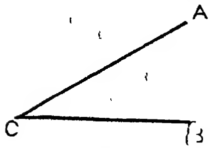


Fig. 1.

Thus the lines AC and BC meet at point C, and form the angle C or ACB or BCA. When three letters are used in naming an angle, the middle one denotes the angular point.

12. The magnitude of an angle depends on the inclination of the two lines to each other; that is to say, upon the size of the opening between them.

Thus the line DC meeting the line AB evidently inclines more to the right than to the left—that is, the angle DCA is greater than the angle DCB; but if CE be drawn so that it shall neither incline to the right nor to the left, thereby making the angle ECB equal to the angle ECA, then each of these angles is called a **right angle** and the line EC is said to be **perpendicular** to the line AB.

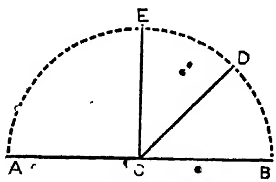


Fig. 2.

The angle DCB is called an **acute** angle, because it is less than a right angle.

The angle DCA is called an **obtuse** angle, because it is greater than a right angle.

13. In the mensuration of surfaces we deal with areas bounded by straight or curved lines, and called **figures**.

14. The boundaries of a figure enclosed by straight lines are called its **sides**.

15. Three-sided figures are called **triangles** because they have three angles or corners; four-sided figures are called **quadrilaterals**; and figures with more than four sides are called **polygons**.

16. All four-sided figures whose opposite sides are equal and parallel to each other are called **parallelograms**.

17. A parallelogram having all its sides equal and all its angles equal is called a **square**.

18. To measure a certain length we compare it with some

definite length fixed upon as a **unit of measurement**, and ascertain the number of times this unit is contained in the given length.

Supposing we wish to find the length of a wall, if the unit of measurement be an inch, the length will be so many *inches*; if the unit of measurement be a foot, the length will be so many *feet*; and so on.

19. The **standard** unit of length is the **imperial yard**, fixed by Act of Parliament to be the distance between two marks on a bar of metal kept in the Exchequer Office.

20. The following table shows the other measures of length and their relation to one another :—

12	inches	make	1 foot.
3	feet	"	1 yard.
$5\frac{1}{2}$	yards	"	1 rod or pole.
40	poles	"	1 furlong.
8	furlongs or	"	1 mile.
1760	yards	"	1 mile.

Since the above measures refer exclusively to the measurement of *lines*, they are termed *linear*.

21. To measure an area we compare it with some **unit of area**, and ascertain the number of times it contains this unit.

22. The unit selected for this purpose is the space included by the lines forming a square. When each side of a square is an inch in length, the space enclosed is a *square inch*; similarly, when each side of a square measures a foot, the space enclosed is a *square foot*, and so on.

23. The **standard** unit of area is a square, each of whose sides is a yard in length. It is therefore called a **square yard**.

24. Let the figure *ABCD* be a square, of which each side represents a yard in length. Divide *AB* into three equal parts; then each of these parts represents a foot in length. Similarly, if *BD* be divided into three equal parts, each of these parts represents a foot in length. Draw parallel lines through the divisions in the sides *AB* and *BD*. Then the square is divided into nine equal squares, each of which is a square foot; thus we see that nine square feet make a square yard. In like manner it may be shown that 144 square inches make one square foot.

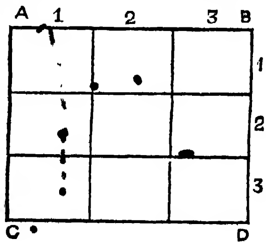


FIG. 3.

25. Be careful to distinguish between **square feet** and **feet square**. For instance, we have just seen that the expression 9 square feet means an area which is equal to a square whose side measures 3 feet (*i.e.* the square is 3 feet square). The expression 9 feet square denotes a square whose side is 9 feet long and whose area is consequently 81 square feet.

26. We see from the demonstration (§ 24) the different units used in the measurement of areas follow at once from the relations between the corresponding units in linear measure.

For since 12 inches make a foot;
$\therefore$ (12×12) square inches make 1 square foot. (1.)
Since 3 feet make a yard;
$\therefore$ (3×3) square feet make 1 square yard. (2.)
And $5\frac{1}{2}$ yards make 1 pole;
$\therefore$ $(5\frac{1}{2} \times 5\frac{1}{2})$ square yards make 1 square pole. (3.)

27. The following table shows these and certain other measures of area :—

144	square inches	make	1 square foot.
9	square feet	,,	1 square yard.
$30\frac{1}{4}$	square yards	,,	1 square rod, pole, or perch.
40	square poles	,,	1 rood.
4	roods	,,	1 acre.
or 4840	square yards	,,	1 acre.
640	acres	,,	1 square mile.

The area of a room is usually expressed in square yards or square feet; the area of land in acres, roods and perches; and the area of a country in square miles.

28. In measuring **land**, surveyors use a chain, known as **Gunter's chain**, which is 22 yards long. This chain is divided into 100 equal parts called **links**; so that each link is $\frac{22}{100}$ of a yard long, or $\frac{66}{100}$ of a foot, or 7·92 inches; for a chain = 100 links = 792 inches = 66 feet = 22 yards = 4 poles.

Example.—Express $4\frac{1}{2}$ miles in chains.
Since $4\frac{1}{2}$ miles $= (1760 \times 4\frac{1}{2})$ yards;
and 1 chain = 22 yards;

$$\therefore 4\frac{1}{2} \text{ miles} = \frac{1760 \times 4\frac{1}{2}}{22} \text{ chains.}$$
$$= 360 \text{ chains.}$$

EXERCISE 1.

1. Express 16 chains 23 links in links.
2. Express 23 chains 5 links in links.
3. Express 10·25 chains in links.
4. Express 36·04 chains in links.
5. Express 4350 links in chains and links.
6. Express 6605 links in chains and links.
7. Express 1529 links in chains.
8. Express 2750 links in chains.
9. Express 4303 links in chains.
10. Express 28 chains in yards.
11. Express 4·25 chains in yards.
12. Express 550 links in yards.
13. Express 138 yards in chains and yards.
14. Express 13440 yards in chains and yards.
15. Express 2860 chains in miles.
16. Express 30000 links in miles.
17. Express 9 miles 4 furlongs 20 poles in chains.
18. Express 5 miles 1 furlong 20 poles in chains.

29. The area of land is also estimated in **square links** and **square chains**, which are connected with the square yard through their linear relations.

Since 22 yards make 1 chain;
∴ (22×22) square yards $= 484$ square yards make 1 square chain;
∴ 4840 square yards $= 10$ square chains $= 1$ acre. (1)
Also, 100 links make 1 chain;
∴ (100×100) square links $= 10000$ square links make 1 square chain.
∴ 100000 square links make 1 acre. (2)

30. To reduce square links and square chains to acres, roods and perches; and conversely.

Example 1.—A surveyor estimates the area of a field to be 19 square chains 1250 links; express this area in acres, roods and perches.

19 chains $= 190000$ links;
19 chains 1250 links $= 191250$ links.
Since 100000 links $= 1$ acre;

$$\therefore \text{The area} = \frac{191250}{100000} \text{ acres.}$$
$$= 1{\cdot}9125 \text{ acres.}$$
$$= 1 \text{ acre } 3 \text{ roods } 26 \text{ perches.}$$

31. Hence, **to convert square links into acres is simply to**

remove the decimal point in the number of square links five places to the left.

Example 2.—Express 9 acres 1 rood 36 perches in acres, chains and links.

9 acres 1 rood 36 perches = 9 acres 76 perches.

Since 160 perches = 1 acre;

$\therefore$ 76 perches $= \frac{76}{160}$ acres.

And 1 acre = 100000 links;

$\therefore \frac{76}{160}$ acres $= \frac{76}{160}$ of 100000 links.

= 47500 links.

= 4 chains 7500 links.

$\therefore$ 9 acres 1 rood 36 perches = 9 acres 4 chains 7500 links.

EXERCISE 2.

1. Express 391250 square links in acres, roods and perches.
2. Express 2467 square chains in acres, roods and perches.
3. Express 56 chains 6875 links in acres, roods and perches.
4. Express 79 chains 8750 links in acres, roods and perches.
5. Express 43 chains 3125 links in acres, roods and perches.
6. Express 7658 chains 625 links in acres, roods and perches.

EXERCISE 3.

1. Express 1 rood 9 perches in chains and links.
2. Express 3 acres 1 rood 36 perches in acres, chains and links.
3. Express 7 acres 2 roods 4 perches in acres, chains and links.
4. Express 10 acres 2 roods 27 perches in acres, chains and links.
5. Express 12 acres 3 roods 18 perches in acres, chains and links.
6. Express 25 acres 3 roods 26 perches in acres, chains and links.

CHAPTER II.

RECTANGULAR PARALLELOGRAMS.

32. **A rectangular parallelogram** is a four-sided figure which has its opposite sides equal and parallel, and all its angles right angles. It is also called a **rectangle.**

A **square** is a rectangle with *all* its sides equal.

A rectangle, of which the opposite sides *only* are equal, is often called an **oblong.**

33. To find the area of a rectangle when the length and breadth are given.

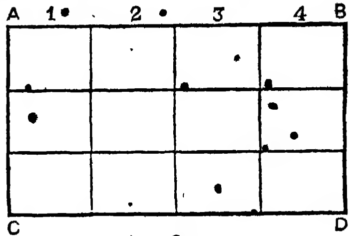


Fig. 4.

Let $ABCD$ be the rectangle whose area is required. AB and CD are called the length, and AC and BD are called the breadth. Suppose that AB is 4 inches and that BD is 3 inches. Divide AB into four equal parts and BD into three equal parts. Draw parallel lines through the divisions in the sides AB and BD. Then the rectangle is divided into twelve equal squares, each of which is a square inch. The number of these squares is clearly the product of the number of inches in AB by the number of inches in BD. In this case the area$=(4 \times 3)$ square inches$=12$ square inches. Hence **the area of a rectangle is found by multiplying the number of inches (or feet, etc.) in its length by the number of inches (or feet, etc.) in its breadth.**

NOTE.—The area of a square is evidently the square of the number of inches (or feet, etc.) in its length.

EXERCISE 4.

Find the areas of the rectangles (in square feet) whose lengths and breadths are as under :—

1. 16 ft., 14 ft.
2. 21 ft., 17 ft.
3. 35 ft., 23 ft.
4. 42 ft., 36 ft.
5. 53 ft., 48 ft.
6. 65 ft., 52 ft.
7. 76 ft., 61 ft.
8. 89 ft., 74 ft.
9. 92 ft., 85 ft.
10. 112 ft., 65 ft.
11. 129 ft., 84 ft.
12. 213 ft., 105 ft.

EXERCISE 5.

Find the areas of the following squares (in square feet) the sides of which are respectively :—

1. 15 ft.
2. 27 ft.
3. 45 ft.
4. 51 ft.
5. 63 ft.
6. 84 ft.
7. 105 ft.
8. 237 ft.
9. 338 ft.

34. Both length and breadth must be expressed in measures of the same name; the product will then give the area in terms of the corresponding square measure.

For example; if the length of a rectangle were 4 feet and the breadth 3 inches, the area of the rectangle would be (48×3) square inches, or $(4 \times \frac{1}{4})$ square feet.

Example.—Find the area of a rectangle whose length is 5 ft. 6 in., and breadth 3 ft. 9 in.

(i.) By **Reduction.** 5 ft. 6 in. = 66 in.; 3 ft. 9 in. = 45 in.
∴ The area = (66 × 45) sq. inches.
= 2970 sq. inches.
= 20 sq. ft. 90 sq. in.

(ii.) By **Fractions.** 5 ft. 6 in. = $5\frac{1}{2}$ ft.; 3 ft. 9 in. = $3\frac{3}{4}$ ft.
∴ The area = $(5\frac{1}{2} \times 3\frac{3}{4})$ sq. feet.
= $20\frac{5}{8}$ sq. feet.
= 20 sq. ft. 90 sq. in.

(iii.) By **Decimals.** 5 ft. 6 in. = 5·5 ft.; 3 ft. 9 in. = 3·75 ft.
∴ The area = (5·5 × 3·75) sq. feet.
= 20·625 sq. ft.
= 20 sq. ft. 90 sq. in.

EXERCISE 6.

Find the areas of the rectangles (in square feet and square inches), whose lengths and breadths are as under:—

1. 15 in., 13 in.
2. 26 in., 19 in.
3. 15 ft. 3 in., 12 ft.
4. 16 ft. 2 in., 14 ft.
5. 18 ft. 9 in., 10 ft.
6. 24 ft., 20 ft. 6 in.
7. 73 ft., 34 ft. 8 in
8. 82 ft., 26 ft. 7 in.
9. 10 ft. 8 in., 6 ft. 8 in.
10. 14 ft. 6 in., 12 ft. 7 in.
11. 48 ft. 6 in., 3 ft. 4 in.
12. 36 ft. 8 in., 7 ft. 11 in.
13. 98 ft. 3 in., 5 ft. 6 in.
14. 141 ft. 6 in., 6 ft. 10 in.

EXERCISE 7.

Find the areas of the rectangles (by decimals) whose lengths and breadths are as under:—

1. 18 ft. 9 in., 14 ft.
2. 18 ft., 5 ft. 9 in.
3. 6 ft. 9 in., 4 ft. 6 in.
4. 5 ft. 9 in., 1 ft. 9 in.
5. 9 ft. 3 in., 7 ft. 6 in.
6. 7 ft. 6 in., 5 ft. 9 in.

EXERCISE 8.

Find the areas of the following squares (in sq. ft. and sq. in.), the sides of which are respectively :—

1. 17 in.	4. 14 ft. 6 in.	7. 29 ft. 2 in.
2. 29 in.	5. 18 ft. 9 in.	8. 32 ft. 4 in.
3. 52 in.	6. 26 ft. 3 in.	9. 36 ft. 7 in.

EXERCISE 9.

Find the areas of the rectangles (in sq. yds., sq. ft. and sq. in.) whose lengths and breadths are as under :—

1. 15 ft., 12 ft.	7. 25 ft. 6 in., 18 ft.
2. 39 ft., 21 ft.	8. 15 ft. 6 in., 12 ft. 9 in.
3. 14 ft., 10 ft.	9. 3 yds. 2 ft., 5 yds. 1 ft.
4. 23 ft., 13 ft.	10. 20 ft. 3 in., 12 ft. 8 in.
5. 21 yds. 2 ft., 18 yds.	11. 14 yds. 9 in., 11 ft.
6. 10 yds., 23 ft.	12. 23 ft. 3 in., 8 ft. 6 in.

EXERCISE 10.

Find the areas of the following squares (in sq. yds., sq. ft. and sq. in.), the sides of which are respectively :—

1. 9 ft. 6 in.	3. 24 ft. 10 in.	5. 13 yds. 9 in.
2. 19 ft. 6 in.	4. 3 yds. 2 ft. 6 in.	6. 30 yds. 2 ft. 9 in.

EXERCISE 11.

1. Find the area of a paving-stone which measures 5 ft. 3 in. by 2 ft. 8 in.

2. What is the area of a square table whose side is 4 ft. 4 in. long ?

3. A room measures 21 ft. 5 in. by 16 ft. 6 in. ; what is the area of the floor ?

4. How many square yards of matting will be required to cover a floor 16 ft. 6 in. long and 12 ft. broad ?

5. How much oil-cloth will it take to cover a table 8 ft. long and $2\frac{1}{2}$ ft. broad ?

6. A wall in a room is 15 ft. 7 in. long and 12 ft. 3 in. high ; what is its area ?

7. A garden is 71 ft. long and 63 ft. wide ; how many square yards does it cover ?

8. A court-yard measures 111 ft. 6 in. by 17 ft. 9 in. ; what is its area in square yards ?

9. A plank of wood is $12\frac{1}{2}$ feet long and 9 inches wide ; express its area in square feet.

10. The length of a building is 47 ft. 3 in., and its breadth is 29 ft. 9 in. ; find the floor-space in the building, if it consist of a basement and three other floors.

11. What is the difference between one area of $3\frac{1}{4}$ feet square, and another of $3\frac{1}{4}$ square feet ?

Example.—Find the acreage of a rectangular field whose length is 125 yards and whose breadth is 121 yards.

$$\begin{aligned}\text{The area} &= (125 \times 121) \text{ sq. yards.} \\ &= 15125 \text{ sq. yards.} \\ &= \frac{15125}{4840} \text{ acres.} \\ &= \frac{25}{8} \text{ acres.} \\ &= 3 \text{ acres } 20 \text{ perches.}\end{aligned}$$

EXERCISE 12.

Find the acreage of the rectangular fields whose lengths and breadths are as under :—

1. 440 yards, 154 yards.
2. 176 yards, 150 yards.
3. $82\frac{1}{2}$ yards, 77 yards.
4. 330 yards, $115\frac{1}{2}$ yards.
5. $797\frac{1}{2}$ yards, 22 yards.
6. 200 yards, 300 feet.

EXERCISE 13.

Find the acreage of the square fields, the sides of which are respectively :—

1. 440 yards.
2. 110 yards.
3. 36 poles.
4. 99 yards.
5. 273 feet.
6. 770 feet.

Example.—Find the acreage of a rectangular field whose length is 26 chains 25 links and whose breadth is 16 chains 5 links.

26 ch. 25 links = 2625 links ; 16 ch. 5 links = 1605 links.

$$\begin{aligned}\therefore \text{The area} &= (2625 \times 1605) \text{ square links.} \\ &= 4213125 \text{ square links.} \\ &= \frac{4213125}{100000} \text{ acres.} \\ &= 42{\cdot}13125 \text{ acres.} \\ &= 42 \text{ acres } 0 \text{ roods } 21 \text{ perches.}\end{aligned}$$

EXERCISE 14.

Find the areas of the rectangular fields (in acres, roods and perches) whose lengths and breadths are as under :—

1. 6 chains, 3 chains 50 links.
2. 9 chains 75 links, 4 chains 50 links.
3. 12 chains 25 links, 8 chains 5 links.
4. 24 chains 68 links, 12 chains 50 links.
5. 20 chains, 2 chains 25 links.
6. 26 chains 50 links, 6 chains 25 links.
7. 15 chains 10 links, 8 chains 75 links.
8. 7 chains 19 links, 6 chains 25 links.
9. 6 chains 25 links, 5 chains 40 links.
10. 18 chains 75 links, 12 chains 50 links.

EXERCISE 15.

Find the areas of the square fields (in acres, roods and perches), the sides of which are respectively :—

1. 850 links.
2. 1250 links.
3. 32 ch. 50 links.
4. $10\frac{1}{2}$ chains.
5. 23 ch. 75 links.
6. 35 ch. 25 links.

DUODECIMALS.

The word *duodecimal* is derived from the Latin word *duodecim*, meaning *twelve.*

35. In previous exercises, where the dimensions were given in different denominations, it was necessary to express both in the same denomination, before calculating the area of a rectangle (or square). In practical work, glaziers, painters, builders, etc., are able to compute the area of a door, floor, ceiling, wall, etc., at once, without changing the denomination of either of the dimensions, by dividing and subdividing the foot into twelfths ; thus—

1 foot $=12$ primes (12′) marked′
1 prime $=12$ seconds (12″) marked″
1 second $=12$ thirds (12‴) marked‴
1 third $=12$ fourths (12^{iv}) marked^{iv}

Each foot, whether it be a linear foot, a square foot, or a cubic foot, is similarly divided ; therefore 12 superficial primes $=$ a square foot and 12 solid primes $=$ a cubic foot.

36. It has been stated that it can be demonstrated that a square foot $=144$ square inches (§ 24). Let AB be the side of a square, and let it represent one foot. Let AB be divided into 12 equal parts, then each part will represent one inch. Through the points of division draw straight lines parallel to AC; then the square is divided into 12 equal rectangles, each 1 foot long and 1 inch broad, and rectangle $ACEF$ is equal to one-twelfth (or superficial prime) of a square foot. Similarly, if AE be divided into 12 equal parts, each part is equal to one 144th (or superficial second) of a square foot.

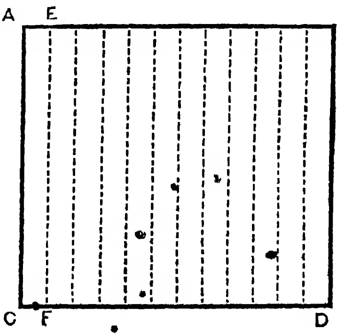


FIG. 5.

37. It being clearly understood that the words *feet*, *inches*, *etc.*, are used to denote the *number* of feet, inches, etc., in the length and breadth of a rectangle, we may say that

(1) Inches × inches = square inches.
(2) Feet × feet = square feet.
(3) Feet × inches = $\frac{1}{12}$ths of a square foot or primes.
(4) Feet × seconds = $\frac{1}{144}$ths of a square foot or seconds.

38. To find the area of a rectangle by duodecimals.

Example 1.—What is the area of a rectangle 3 feet 5 inches long, and 2 feet 4 inches broad?

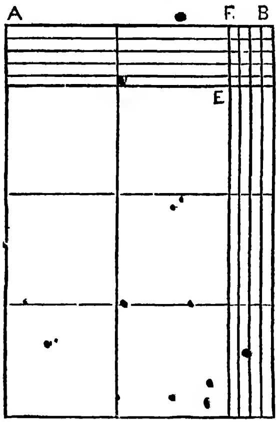


FIG. 6.

Length	3 ft.	5 in.	
Breadth	2	4	
	6	10	
	1	1	8
	7	11′	8″

The area is to be found by multiplying the number of feet and inches in the length by the number of feet and inches in the breadth (§ 33).

In the diagram, let the line *AB* represent 2 ft. 4 in., and the line *BC* 3 ft. 5 in. The whole area *AC* is made up of pieces of different sizes.

The six largest pieces measure 1 foot each way, and therefore represent six square feet. They form the figure *DE*.

Other pieces measure 1 foot one way and 1 inch the other. Each of these contains one-twelfth of a square foot (a prime). There are 10 of these in figure *AE*, and 12 in the figure *EC*.

The twenty smallest pieces measure 1 inch each way, and therefore represent 20 square inches. They form the figure *EB*.

The whole area is therefore seen to be 6 feet 22 primes 20 inches; but since 12 of the smallest pieces make 1 prime; and 12 primes make 1 foot, the result = 7 square feet 11 primes 8 square inches, or 7 square feet 140 square inches.

We will explain the method by another example.

Example 2.—What is the area of a rectangle 11 feet 5 inches long, and 7 feet 6 inches wide.

Length	= 11 ft.	5′		
Breadth	= 7 ft.	6′		
	79	11′		= 11 ft. 5 in. × 7 ft.
	5	8′	6″	= 11 ft. 5 in. × 6 in.
Area	= 85 sq. ft.	7′	6″	

Having placed feet under feet, and inches under inches, multiply each term of the multiplicand by the number of feet in the multiplier, beginning at the right hand. Thus, 7 times 5′ are 35′=2 sq. ft. and 11′. Set down 11′, and carry 2 sq. ft. to the next product. Proceeding, 7 times 11 are 77, and adding the 2 carried we have 79 sq. ft. Next, multiply by 6′. Since 6′=$\frac{6}{12}$ of a foot, and 5′=$\frac{5}{12}$ of a foot, 5′×6′=$\frac{30}{144}$ of a square foot=30″=2′ and 6″. Write the 6″ one place to the right of the primes, and carry 2′ to the next product. Then the product of 6′ or $\frac{6}{12}$ of a foot and 11 ft.=$\frac{66}{12}$ of a square foot, or 66′, which, added to the 2′ carried=68′ =5 sq. ft. and 8′. Place the 8′ under the 11′ and the 5 sq. ft. under the 7 sq. ft. Now, adding these products, the sum is 85 sq. ft. 7′ 6″.

NOTE.—In square measure the inch is $\frac{1}{144}$ of a foot, while a prime is $\frac{1}{12}$ of a foot, or 12 square inches. The true value of the above result is therefore 85 sq. feet, 7 twelfths of a square foot and 6 square inches. The second and third denominations can at once be combined into square inches, by multiplying the second by 12 and adding the third; therefore 85 sq. feet 7′ 6″=85 sq. feet 90 sq. inches.

EXERCISE 16.

Express in duodecimals :—

1. 3 ft. 6 in.
2. 7 ft. 5½ in.
3. 10 ft. 9¼ in.
4. 15 sq. ft. 25 sq. in.
5. 46 sq. ft. 53 sq. in.
6. 110 sq. ft. 116 sq. in.

EXERCISE 17.

Express in feet and inches :—

1. 6 ft. 7′.
2. 25 ft. 8′ 4″.
3. 47 ft. 2′ 9″.
4. 4 sq. ft. 3′ 5″.
5. 11 sq. ft. 9′ 0″ 6‴.
6. 83 sq. ft. 6′ 8″ 3‴.

EXERCISE 18.

Express each answer in square feet and square inches.

Find, by duodecimals, the areas of the rectangles whose lengths and breadths are as under :—

1. 3 ft. 2 in., 5 ft. 7 in.
2. 9. ft. 8 in., 7 ft. 6 in.
3. 18 ft. 4 in., 14 ft. 3 in.
4. 17 ft. 3 in., 14 ft. 7 in.
5. 32 ft. 8 in., 18 ft. 4 in.
6. 38 ft. 5 in., 9 ft. 11 in.
7. 10 ft. 1′ 6″, 7 ft. 6′.
8. 23 ft. 5′ 4″, 17 ft. 9′.
9. 11 ft. 1′ 6″, 6 ft. 6′ 8″.
10. 13 ft. 8′ 11″, 14 ft. 9′.
11. 10 ft. 5′ 3″, 9 ft. 7′ 10″.
12. 37 ft. 5′ 6″, 22 ft. 7′ 8″.

39. Since the area of a rectangle is found by multiplying its length by its breadth, using the words *length, breadth* and *area* to denote the number of units in each respectively, it follows that if the area be known and also the length (or the breadth),

the breadth (or the length) can be at once found. This can be expressed briefly thus:—

$$\text{Since} \quad \textbf{length} \times \textbf{breadth} = \textbf{area};$$

$$\text{therefore} \quad (1)\ \textbf{length} = \frac{\textbf{area}}{\textbf{breadth}},$$

$$\text{and} \quad (2)\ \textbf{breadth} = \frac{\textbf{area}}{\textbf{length}}.$$

Example.—Find the length of a rectangle whose breadth is 3 yards 2 feet, and whose area is 19 square yards 5 square feet.

19 sq. yds. 5 sq. ft. = 176 sq. ft. ; 3 yds. 2 ft. = 11 ft.
∴ Length = (176 ÷ 11) ft. = 16 ft. = 5 yds. 1 ft.

NOTE.—1. Both the area and the given side must be expressed in the same denomination before the process of division.

2. The length of the side required will be expressed in the same denomination.

EXERCISE 19.

Find the length of the following rectangles, having given the area and the breadth :—

1. Area, 975744 sq. ft. ; breadth, 528 ft.
2. Area, 633 sq. ft. 135 sq. in. ; breadth, 17 ft. 3 in.
3. Area, 76 sq. yds. 6 sq. ft. ; breadth, 2 yds. 5 ft.
4. Area, 6 acres 363 sq. yds. ; breadth, 99 yds.
5. Area, 8 acres 1 rood 10 perches ; breadth, 665 links.
6. Area, 87 acres 1 rood 26 perches ; breadth, 9 chains 25 links.

EXERCISE 20.

Find the breadth of the following rectangles, having given the area and the length :—

1. Area, 23 sq. ft. 72 sq. in. ; length, 7 ft. 10 in.
2. Area, 35 sq. yds. 7 sq. ft. 126 sq. in. ; length, 20 ft. 6 in.
3. Area, 177 sq. yds. 5 sq. ft. ; length, 15 yds. 2 ft.
4. Area, $3\frac{1}{2}$ acres ; length, 242 yds.
5. Area, 28 acres 22 perches ; length, 4502 links.
6. Area, 16 acres 2 roods 10 perches ; length, 6 chains 25 links.

EXERCISE 21.

1. What length must be cut off a board $6\frac{3}{4}$ inches wide, that the area cut off may be a square foot?

2. Reckoning the length of the Suez Canal to be 60 miles, and its surface equal to a square mile, what is its average width in feet?

3. A row of houses stands upon an acre of ground; the frontage is 300 yards. What is the depth?

4. Find in feet, to two places of decimals, the length of a sheet of lead, 19 inches broad, which covers 233 square feet.

5. A rectangle 25 feet long and 14 feet broad is equal to the area of a rectangle 20 feet long. What is the breadth of the latter?

6. A man in mowing grass by a machine advances at the rate of 4 miles an hour. It takes him 72 minutes to mow a plot covering 10560 square yards. How broad does he mow?

10. It has been shown that the area of a square is the square of its length, because its length and breadth are equal; therefore it follows that if the area be known, *i.e.* the square of the length, the length itself is the square root of the number denoting the area.

Example.—The area of a square field is 3 acres 2 roods 7 perches $9\frac{1}{4}$ square yards; what is the length of each of its sides in yards?

Here area of square = 3 ac. 2 ro. 7 per. $9\frac{1}{4}$ sq. yds. = 17161 sq. yds.

$\therefore$ Side of square = $\sqrt{17161}$ yards = 131 yards.

NOTE.—1. When the area is expressed in different denominations, it must be reduced to its lowest denomination before the square root can be found.

2. Remember that the units in the area are expressed in square measure; those in the side in linear measure.

EXERCISE 22.

Find the length of the sides of the squares whose areas are respectively:—

1. 529 sq. feet.
2. 9604 sq. feet.
3. 64009 sq. feet.
4. 46656 sq. yards.
5. 824464 sq. yards.
6. 552049 sq. yards.
7. 141·61 sq. feet.
8. 19·7136 sq. feet.
9. 2334·8224 sq. feet.
10. $10\frac{6}{25}$ sq. yards.
11. $1\frac{13}{36}$ sq. yards.
12. $110\frac{1}{4}$ sq. yards.

EXERCISE 23.

Find the length of the sides of the squares whose areas are respectively:—

1. 3 sq. ft. 97 sq. in.
2. 5 sq. ft. 121 sq. in.
3. 14 sq. ft. 9 sq. in.
4. 150 sq. feet. 9 sq. in.
5. 600 sq. ft. 36 sq. in.
6. 7367 sq. ft. 52 sq. in.
7. 42 sq. yds. 5 sq. ft. 73 sq. in.
8. 25 sq. yds. 7 sq. ft. 81 sq. in.
9. 8 ac. 16 per.
10. 25 ac. 2 ro. 16 per.
11. 3 ac. 1 ro 13 per. $5\frac{3}{4}$ sq. yds.
12. 109 ac. 3 ro. 8 per. 9 sq. yds.

EXERCISE 24.

1. Find in yards the side of a square plot of land which contains $122\frac{1}{2}$ acres.

2. A plot of ground in the form of a square contains exactly one acre; how long is its side, expressed in links?

3. If a square field contains 10 acres of ground, what is the length of its side in poles?

4. A square lawn contains 1 acre 6 perches $19\frac{1}{2}$ square yards. Find, in yards, the length of its side.

5. The area of a square park is 115 acres 2 roods 16 perches. Find the length of its side in chains.

6. The area of a square field is equal to the area of a rectangle whose sides are respectively 21 ft. 11 in. and 197 ft. 3 in. Find the length of the side of the square.

41. The distance around the boundary of a figure is called its **perimeter.** The perimeter of a parallelogram is therefore the sum of the lengths of its sides.

Example.—A square court-yard contains 8649 square feet; what is its perimeter?

Here area of square $= 8649$ sq. ft.

$\therefore$ Length of one side $= \sqrt{8649}$ ft. $= 93$ ft.

$\therefore$ Perimeter $= (93 \times 4)$ ft. $= 372$ feet.

EXERCISE 25.

1. How many yards of fencing are required to enclose a square field containing 1936 sq. yards?

2. If a square plot of ground contain 961 square yards, how many palings placed a yard apart will be required to go round it?

3. A square field covers 57 acres 2 roods 16 perches. What is its perimeter in yards?

4. The side of a square field measures 20 chains 25 links. If I walk round the boundary, how many miles shall I travel?

5. A square field contains 21 acres 3 roods 1 perch. How long will it take a man to run round the boundary at the rate of $7\frac{1}{2}$ miles an hour?

6. Six times round a cricket-ground measures nine miles. How many chains are there in one side, if the field be a perfect square?

42. When the length of a rectangle is a multiple of its breadth, the entire surface of the rectangle can be divided into squares, each of whose sides will be equal to the breadth of the rectangle.

Example.—A room is twice as long as it is broad, and its area measures 338 square yards; find its dimensions.

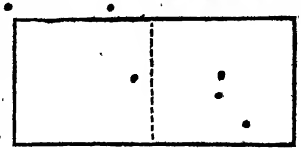


Fig. 7.

Since the length is twice the breadth, the room can be divided into two squares, each of whose sides is equal to the breadth of the room.

Here area of each square = 169 sq. yards.

∴ Side of each square = $\sqrt{169}$ = yds. 13 yds.

∴ Breadth of the room = 13 yards.

∴ Length of the room = 26 yards.

Note.—The pupil should draw diagrams in connexion with the solution of most of the Exercises.

EXERCISE 26.

1. A room is twice as long as it is broad, and its area is 2312 square feet. Find the length of each side.

2. A tennis court is twice as long as it is broad, and its area is 56 square yards 8 square feet. Find the length of each side.

3. A room whose length is three times its breadth has an area of 581 square feet 3 square inches. What are its dimensions?

4. The area of a rectangular field is 39 acres 32 perches, and one side is double the other. What are its dimensions in yards?

5. The area of a rectangular field is 22 acres 2036 square yards, and one of the sides is three-and-a-half times as long as the other. Find its dimensions in yards.

6. A floor is half as long again as it is broad, and its area is 13824 square feet. Find the length of each side.

43. To find the area of a uniform path.

Example.—A tennis court 58 feet long and 34 feet wide has a gravel path 2 feet wide outside it. Find the area of the path.

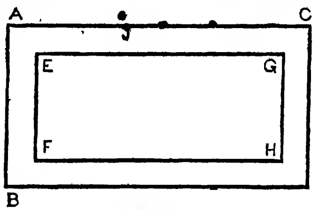


Fig. 8.

Let figure *EFGH* represent the court, then it is evident that the figure *ABCD* includes the court and the path; therefore the difference of these two areas must be the area of the path.

To obtain the length and breadth of the outer rectangle *ABCD* twice the width of the path must be added to the length and breadth of the inner rectangle *EFGH*.

Hence area of rectangle $ABCD$ = (62 × 38) square feet.
= 2356 square feet.

Area of rectangle $EFGH$ = (58 × 34) square feet.
= 1972 square feet.

∴ Area of path = (2356 − 1972) square feet.
= 384 square feet.

Note.—If the path had been *inside* the court it would have been necessary to *subtract* twice the width of the path from the dimensions of the court.

EXERCISE 27.

1. A square tennis court whose side is 60 feet long, has a gravel path 3 feet wide outside it. Find the area of the path.

2. A square plot of ground is 50 feet long. If a gravel path 3 feet wide be taken off all round, find the area of the path.

3. A park is 908 yards square. If a drive 42 feet wide be made all round it inside, what will be its area in square yards?

4. An oblong field is 300 yards long and 200 yards broad. If a belt of trees 30 yards wide be planted round it, find the area of the interior space in square yards.

5. The outer and inner boundaries of a gravel path are squares, and the path is 9 feet wide. The square enclosed by the path covers 304 acres 9 square yards. Find the area of the path.

6. A rectangular field 12 chains long and $7\frac{1}{2}$ chains broad has a square pond in the middle whose side measures $12\frac{1}{2}$ yards. Find the area of the grass land in acres, roods and perches.

7. A square field has a plantation 11 yards wide running along all four sides without the boundary of the field. This border contains one acre. Find the area of the field.

8. A rectangular park, one side of which is twice as long as the other, contains 500 acres. How many square yards of ground will be occupied by a road 15 feet wide running round it?

44. To find the cost of an area at a given price.

It is often necessary, not only to find the area of a floor, court, field, etc., but also the cost of the workman's labour at so much per sq. foot, sq. yard, etc., or the rent or value (in the cost of land) at so much per acre, etc.

EXERCISE 28.

1. What must be paid for paving a court measuring 2344 sq. feet 72 sq. inches at 5s. 6d. per square yard.

2. Find the cost of turfing a lawn measuring 1 acre 6 perches $19\frac{1}{2}$ square yards at 3d. per square foot.

3. If building-ground be bought for 15s. 9d. a square yard, what will be the cost of half-an-acre of such ground?

4. Find the rent of a farm of 225 acres 1 rood 38 perches at £2 11s. 8d. per acre.

5. Find the cost of four building sites, each 3 roods 23 perches, at £278 15s. 10d. per acre.

6. What is the cost of mowing a meadow whose area is 29 acres 2 roods 35 perches at 3s. 6d. per acre?

7. Find the cost of turfing a lawn 140 feet long and 41 feet broad at $6\frac{3}{4}$d. per square yard.

8. A square playground is 17 yards 2 feet long. What will be the expense of gravelling the same, at 1s. $1\frac{1}{2}$d. per square yard?

9. Find the cost of levelling a rectangular plot 120 feet long and 60 feet broad at £6 1s. per square chain.

10. Find the value of a building site 79 feet 6 inches long and 52 feet 8 inches wide at 3s. per square foot.

11. What is the value of a meadow 16 chains 50 links long and 3 chains 50 links broad at £20 per acre?

12. Find the cost of gravelling a footpath $\frac{1}{4}$ mile in length and $4\frac{1}{2}$ feet wide at a cost of 5d. per square yard.

EXERCISE 29.

1. Find the cost of turfing a lawn 30 yards 2 feet 6 inches long and 10 yards 1 foot broad at 2s. 3d. per square yard; two flower-beds 10 feet long by 8 feet broad are not to be turfed.

2. In a rectangular pleasure ground, 96 feet by 84 feet, there are four oblong grass plots, $22\frac{1}{2}$ feet by 18 feet each. Find the cost of asphalting the remaining portion of the ground at $8\frac{1}{4}$d. per square yard.

3. What will it cost to make a gravel walk 7 feet wide along the inner edge of each side of a square field whose side is 110 yards long at 1s. 6d. per square yard?

4. A square pond whose side is 75 feet has a path $2\frac{1}{2}$ yards wide round and immediately outside. Find the cost of asphalting the path at 9d. per square yard.

5. Find the expense of paving a pathway 6 feet wide round and immediately outside a tennis court 21 yards long and 10 yards wide at $9\frac{1}{4}$d. per square yard.

6. Find the cost of gravelling a path 5 feet wide running along two consecutive sides of a garden which is 172 feet long and 54 feet wide; a load of gravel, worth 12s. 9d., being required for every 40 square yards of path, and the work occupying a man and a boy for $4\frac{1}{2}$ days at 3s. 9d. and 2s. a day respectively.

7. A rectangular pleasure ground is 100 feet long and 60 feet broad. Two paths cross it at right angles, one from end to end, and the other from side to side. Each of these is 5 feet wide. Find the cost of laying down the remaining area with turf at 6d. per square yard.

8. A rectangular court is 120 feet long and 90 feet broad, and a path of the uniform breadth of 10 feet runs round it. Find the cost of paving the path at 4s. 6d. per square yard, and covering the remainder of the court with turf at 6s. 6d. per 100 square feet.

45. To find how many times one area is contained in another area.

Example.—A railway platform is 54 yards long and 21 yards broad. How many planks does it contain, each being $13\frac{1}{2}$ feet long and $10\frac{1}{2}$ inches wide?

Here area of platform $= (54 \times 21)$ sq. yards $= 1134$ sq. yards.

$\therefore$ Number of square inches in platform $= 1134 \times 9 \times 144 = 1469664$.

But the number of square inches in one plank $= 13\frac{1}{2} \times 12 \times 10\frac{1}{2} = 1701$.

$\therefore$ Number of planks required $= \frac{1469664}{1701} = 864$.

NOTE.—1. The areas must be both expressed in the same denomination.
2. The quotient obtained by dividing the greater space by the less is the number required.

EXERCISE 30.

1. How many allotments each $1\frac{1}{2}$ perches can be made from a field covering $6\frac{3}{4}$ acres?

2. How many sheets of paper each containing 54 square inches will cover an acre of ground?

3. A dining-hall whose area is 810 square feet is to be paved with rectangular tiles each 12 inches by 9 inches. How many tiles will be required?

4. A lawn 90 feet long and 37 feet broad is to be laid with turfs each 9 inches square. How many will be required?

5. Into how many allotments each containing 2 roods 10 perches can a rectangular field 450 yards long and 121 yards wide be divided?

6. How many oblong tiles $4\frac{1}{2}$ inches by $2\frac{1}{2}$ inches will be required for paving a square court each side of which measures 12 feet 6 inches?

7. How many deals 4 feet long and 8 inches wide are required for the floor of a room 16 feet long and 12 feet 9 inches wide?

8. Find the number of granite blocks required to pave a street 1 mile long and 16 yards wide, one block being 4 inches broad and 12 inches long.

9. Find the cost of turfing a ground 10 chains long and 5 chains broad each turf being 15 inches by 6 inches and 100 turfs costing 1s. 3d.

10. How many tiles $11\frac{3}{4}$ inches by $7\frac{1}{2}$ inches will be required to pave a yard 18 feet 9 inches by 15 feet 8 inches? And what will be the cost, if the tiles be 9d. per dozen and the expense of laying them down 9d. per square yard?

11. The length of a rectangular field is 209 yards and its width is 110 yards. How many allotments of $\frac{1}{4}$ acre can be made from it? And how much more rent will be got by letting the allotments at 10s. each than by letting the whole field at 30s. per acre?

12. Taking the cost of flagstones at 1s. per square foot, what sum must be given for stones, each 14 inches by 9 inches sufficient to pave a certain court-yard, if a court-yard of half the size require 560 stones each $1\frac{1}{2}$ feet square?

EXERCISE. 31.

1. If the cost of paving a square court be £1 7s. at the rate of 1s. 4d. per square yard, how many yards long is the side of the court?

2. If the cost of making a square lawn be £351 18s. $4\frac{1}{2}$d. at the rate of 3s. $4\frac{1}{2}$d. per square yard, how many feet long is the side of the lawn?

3. The expense of paving a street half-a-mile long, at $7\frac{1}{2}$d. per square yard, was £430 16s. 8d. Find the breadth of the street.

4. The cost of gravelling a rectangular playground, at 9d. per square foot, was £17 8s. 10d. If one side measure 30 feet 4 inches, what is the length of the other side?

5. The rent of a square field, at £2 14s. 6d. per acre, amounts to £27 5s. Find the cost of surrounding the field with a railing at 9d. per yard.

6. The cost of levelling and turfing a square cricket field at £175 9s. 4d. per acre is £987. Find the cost of putting a paling round it at 3s. 2d. per yard.

46. To find the length of carpet required to cover a floor.

Carpet is made in various widths, and is sold by the yard. It is evident (§ 39) that to find the number of yards of carpet required to cover the floor of a room, the area of a room must be divided by the width of the carpet. The number of yards found, the cost can be readily ascertained.

Example.—How much carpet 27 inches wide will be required for a room 31 feet 6 inches long and 16 feet 6 inches broad; and what will it cost at 5s. 6d. per yard?

Area of floor $=(31\frac{1}{2}\times16\frac{1}{2})$ sq. feet $=519\frac{3}{4}$ sq. feet.

Since the carpet is to cover the whole floor, the whole area of the carpet is to be $519\frac{3}{4}$ sq. feet, and its width is $\frac{27}{12}$ feet.

Hence length of carpet required $=(519\frac{3}{4}\div\frac{27}{12})$ feet $=231$ feet $=77$ yards.

And cost of carpet $=$ 5s. 6d. $\times 77=$ £21 3s. 6d.

NOTE.—Similarly, the width of the carpet, the length of the room, the breadth of the room, or the price of the carpet can be found when the other quantities are given.

EXERCISE 32.

1. How many yards of carpet 27 inches wide will cover a room 27 feet long and $17\frac{1}{2}$ feet broad?

2. How many yards of carpet 2 feet 11 inches wide will cover a room 15 feet long and 14 feet 7 inches broad?

3. What length of carpet $\frac{3}{4}$ yard wide will be required to cover a floor $39\frac{1}{2}$ feet by $25\frac{1}{2}$ feet?

4. Find the expense of carpeting a room 18 feet 9 inches long and 17 feet 6 inches broad with carpet 28 inches wide at 5s. 2d. per yard.

5. How much will it cost to carpet a room 12 feet 9 inches by 16 feet 6 inches with carpet 33 inches wide at 3s. 8d. per yard?

6. Find the cost of carpeting a floor 19 feet 7 inches long and 18 feet 9 inches broad with carpet 25 inches wide at 5s. 6d. per yard.

7. When 54 yards 2 feet 6 inches of carpet exactly cover a floor 23 feet 6 inches long and 15 feet 9 inches broad, what is the width of the carpet?

8. A room 21 feet long requires 49 yards of carpet 27 inches wide. Find the breadth of the room.

9. What is the length of a side of a square room which is covered with $206\frac{1}{4}$ yards of carpet 33 inches wide?

10. A square room is covered with carpet 27 inches wide costing 3s. $4\frac{1}{2}$d. per yard. If the cost of the carpet be £15 12s. 6d., find the length of a side of the room.

11. The cost of covering a room with a Turkey carpet at 6s. 3d. per square yard is £7 5s. 10d. If the width of the room be 13 feet 4 inches, find the length of the room.

12. The cost of covering a room with carpet 3 feet 3 inches wide is £6 3s. 6d. The dimensions of the room are 19 feet 6 inches by 13 feet. Find the price of the carpet per yard.

47. The following example should be carefully noticed.

Example.—A room 20 feet by 16 feet has a Turkey carpet in it, a border 2 feet wide all round being left uncovered by the carpet. The border was stained at a cost of 1d. per square foot, and the carpet cost 27s. per square yard. What was the whole expense?

Since the border is 2 ft. wide all round the room, the length of the carpet must be 20 ft. − 2 ft. × 2 = 16 ft., and the breadth must be 16 ft. − 2 ft. × 2 = 12 ft.

Hence area of carpet $= 16 \times 12$ sq. ft. $= \frac{64}{3}$ sq. yds.

Hence price of carpet $=$ 27s. $\times \frac{64}{3} =$ £28 16s.

The area of the border is the difference between the whole area of the floor and the area covered by the carpet, and is therefore

$(20 \times 16 - 16 \times 12)$ sq. ft. $= 128$ sq. ft.

Hence cost of staining at 1d. per sq. foot = 128d. = 10s. 8d.

∴ Total cost = £28 16s. + 10s. 8d. = £29 6s. 8d.

EXERCISE 33.

1. If a carpet measuring 20 feet by 15 feet be laid in a room 24 feet by 18 feet, how many square yards of the floor will remain uncovered?

2. A carpet 13 feet 6 inches by 10 feet 9 inches is laid on a floor 20 feet 6 inches by 12 feet 9 inches. Find the cost of staining the rest of the floor at 3d. per square yard.

3. A room is 22 feet 6 inches long by 13 feet 5 inches wide. Find the cost of staining a border next the walls 18 inches broad all round the room at 2s. per square yard.

4. A room measuring 19 feet by 17 feet is to be laid with matting so as to leave an uncovered piece next the walls 18 inches broad all round the room. How many square yards of matting will be required?

5. What length of carpet 3 feet 6 inches wide will be required for a room 18 feet long and 16 feet broad, if a space a foot broad be left uncovered all round the room?

6. A room 21 feet square has a square of Turkey carpet in the centre, a border 3 feet wide inside all round being covered with oil-cloth. The carpet and oil-cloth cost respectively 16s. 6d. and 8s. 6d. per square yard. Find the total cost.

7. A room 24 feet by 18 feet is covered in the centre with carpet 2 feet wide, at 4s. 3d. per yard, leaving a border of 3 feet all round the carpet, which was painted at a cost of 9d. per square yard. Find the total cost.

8. Find the saving in carpeting a room 18 feet long and 15 feet wide, if, instead of carpeting the whole floor, a width of 3 feet from each wall be stained. The carpet 18 inches wide costs 3s. 9d. per yard, and staining costs 4d. per square foot.

48. To find the area of the surface of the walls of a room.

The area of the four walls of a room is equal to the area of a rectangle, whose length is the distance round the room, and whose breadth is the height of the room; for the area of each of the two sides of a room = length × height and the area of each of the two ends of a room = breadth × height. Hence **the total area of the four walls of a room = (twice the length + twice the breadth) × the height.**

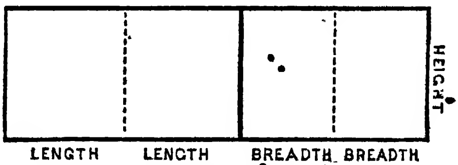


FIG. 9.

Example.—What will be the cost of painting the walls of a room 19 feet long, 15 feet broad and 10 feet high at 2s. 3d. per square yard.

The total area of the four walls in question

$= \{2(19+15) \times 10\}$ square feet

$= 680$ square feet $= 75\frac{5}{9}$ square yards.

Since the painting costs 2s. 3d. per square yard the whole cost will be

2s. 3d. $\times 75\frac{5}{9} =$ £8 10s.

NOTE.—Painters' work is usually estimated by the square yard.

49. Deductions must be made from the total area of the the walls for windows, fire-places, doors and cupboards, before calculating the expense of painting or papering, as the case may be.

Example.—A room is 14 feet 9 inches long, 9 feet 3 inches broad and 10 feet 6 inches high. It contains two windows 5 feet 6 inches by 4 feet, three doors 6 feet by 3 feet and a fire-place $6\frac{1}{2}$ feet long by 4 feet high. What is area of the surface to be painted?

The total area of the four walls in question

$=\{2(14\frac{3}{4}+9\frac{1}{4})\times 10\frac{1}{2}\}$ square feet
$=504$ square feet.

Area of windows	$= (5\frac{1}{2}\times 4\times 2)$ sq. feet	$= 44$ sq. ft.
Area of doors	$= (6\times 3\times 3)$ sq. feet	$= 54$ sq. ft.
Area of fireplace	$= (6\frac{1}{2}\times 4)$ sq. feet	$= 26$ sq. ft.
Total area to be deducted	$= (44+54+26)$ sq. feet	$=124$ sq. ft.
∴ Area to be painted	$= (504-124)$ sq. feet	$=380$ sq. ft.

EXERCISE 34.

1. A room is 18 feet long, 15 feet broad and 13 feet high. Find the area of the walls in square yards.

2. A room is 14 feet 3 inches long, 11 feet 9 inches broad and 10 feet high. Find the area of the walls in square yards.

3. A room is 40 feet long, 30 feet broad and 20 feet high. Find the cost of painting the walls at 2s. 6d. per square yard.

4. What will be the cost of painting the walls of a room at 1s. 9d. per square yard, if the length of each side be 15 feet, of each end 10 feet, and the height $9\frac{3}{4}$ feet?

5. A room is 16 feet long, 8 feet 4 inches broad and 7 feet 6 inches high. Find the cost of painting the walls at $4\frac{3}{4}$d. per square foot.

6. A room is 20 feet $7\frac{3}{4}$ inches long, 15 feet $4\frac{1}{4}$ inches broad and 12 feet 4 inches high. Find the cost of colouring the walls at 9d. per square yard.

7. Find the expense of painting a room 22 feet 6 inches long by 13 feet 5 inches broad and 9 feet high at 4s. per square yard, deducting 10 square yards for windows.

8. Find the expense of painting a room 27 feet long, $17\frac{1}{2}$ feet broad and $11\frac{1}{2}$ feet high at 3s. per square yard, allowing for four windows each $7\frac{1}{2}$ feet by 4 feet.

9. Find the cost of painting a room 21 feet long, 15 feet wide and 11 feet 4 inches high at 1s. 3d. per square yard, allowance being made for two windows 7 feet high by 3 feet wide and a fireplace 6 feet wide by $4\frac{1}{2}$ feet high.

10. The walls of a room which is 35 feet long, $17\frac{1}{2}$ feet wide and 18 feet high, are to be panelled to the height of 4 feet at 5s. 6d. per square foot, and coloured above the panelling at 1s. 6d. per square yard. The doors and windows reduce the area to be panelled by 25 square feet, and that to be coloured by 60 square feet. What will be the cost of the whole work?

50. Wall papers are only sold in lengths of 12 yards called *pieces* and English wall papers are generally 21 inches wide.

Example.—A room is 18 feet long, 13 feet 6 inches wide and 12 feet high. How many pieces of paper (each 12 yards) 21 inches wide will be required? And what will be the cost at 2s. 6d. per piece?

The area of the four walls in question

$=\{2\,(18+13\frac{1}{2})\times 12\}$ square feet $=756$ square feet.

Hence the length of paper required

$=(756\div\frac{21}{12})$ feet $=432$ feet $=144$ yards.

Since each piece contains 12 yards, the number of pieces required

$=\frac{144}{12}=12$.

And the cost $=$ 2s. 6d. $\times 12=$ £1 10s.

EXERCISE 35.

1. How many pieces of wall paper each 12 yards long and 24 inches wide will be required to cover the four walls of a room 16 feet square and 9 feet high?

2. How many pieces of wall paper each 12 yards long and $9\frac{1}{4}$ inches wide must be bought to cover the walls of a room 29 feet 7 inches long, 16 feet 8 inches wide and $13\frac{1}{2}$ feet high?

3. How many yards of paper $\frac{7}{8}$ yard wide will cover the walls of a room 23 feet 6 inches long by 18 feet 6 inches wide and $10\frac{1}{2}$ feet high?

4. How many yards of paper 2 feet wide will be required for a room 35 feet long, 25 feet wide and 10 feet high; allowing for a door 7 feet high and 6 feet wide, two windows each 6 feet high and 4 feet wide and a fireplace covering 18 square feet?

5. How many pieces of wall paper each 12 yards long and 9 inches wide are required to cover the walls of a room 24 feet long, 16 feet wide and $12\frac{1}{2}$ feet high; allowing $\frac{2}{9}$ of a piece for waste, but deducting for door, windows, etc. $\frac{1}{4}$ of the whole area?

6. Find the cost of papering a room 14 feet long, 13 feet 6 inches wide and 7 feet 8 inches high with paper 18 inches wide at 13s. 6d. per piece of 12 yards.

7. Find the cost of paper 27 inches wide for a room 27 feet 8 inches long, 21 feet 4 inches wide and 12 feet high at 4s. 6d. per piece of 12 yards.

8. What will be the expense of papering a room 20 feet long, 15 feet broad and 13 feet high with paper 21 inches wide at 11d. per piece of 12 yards; an extra 8d. per piece being charged for hanging?

9. Find the cost of papering a room 24 feet 10 inches long, 13 feet 5 inches wide and 11 feet 4 inches high with paper 1 foot 11 inches broad costing $7\frac{3}{4}$d. per yard: an area of 108 square feet being deducted for doors and windows.

10. A room is 21 feet 4 inches long, 15 feet 9 inches broad and 14 feet high; the doors and windows together occupy 65 square feet. What will be the cost of papering the remaining part of the surface of the walls with paper 25 inches wide at 3s. 9d. per piece of 12 yards?

11. What will be the cost of papering a room whose length is 28 feet, breadth 20 feet and height 10 feet with paper 16 inches wide at 10s. per piece of 12 yards; allowing for a fireplace 5 feet by 4 feet, a door 7 feet by 4 feet and two windows each 5 feet by 3 feet?

12. A room 25 feet 7 inches long, 16 feet 9 inches broad and 13 feet 6 inches high has three windows each 5 feet by 3 feet 6 inches, a door 7 feet by 4 feet and a fireplace 5 feet by 6 feet. What will be the cost

of covering the remaining part of the surface of the walls with paper $\frac{5}{8}$ of a yard wide at 4s. 6d. per piece of 12 yards?

51. Glaziers usually estimate their work by the square foot.

EXERCISE 36.

1. How many panes of glass each 2 feet by 1 foot 5 inches will glaze a window 8 feet high and 5 feet 8 inches wide?

2. There are 8 windows to be glazed and each measures 1 foot 6 inches by 3 feet. What will be the cost at $7\frac{3}{4}$d. per square foot?

3. Find the cost of glazing 6 windows, each window having 12 panes and each pane measuring 15 feet 6 inches by 8 feet 9 inches, at 8d. per square foot.

4. In a row of twenty-five houses, each house has 17 windows, each window 4 panes, and each pane measures 18 inches by 9 inches. What will be the cost of glazing all these windows at 6d. per square foot?

5. Required the expense of glazing the windows of a house at 1s. 8d. per square foot, there being 8 which measure 5 feet 6 inches by 2 feet 10 inches and 10 which measure 4 feet 8 inches by 2 feet 9 inches.

6. A certain building has 63 windows, 46 of which contain 12 panes each, measuring 20 inches by 16; the others contain 9 panes each, measuring 16 inches square. Find the cost of glazing at 2s. 3d. per square foot.

CHAPTER III.

OBLIQUE PARALLELOGRAMS.

52. A four-sided figure which has all its sides equal, while its angles are not right angles, is called a **rhombus** (A).

53. A four-sided figure which has its opposite sides *only* equal to each other, while its angles are not right angles, is called a **rhomboid** (B).

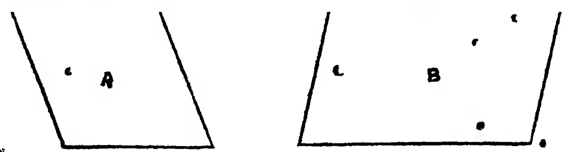


FIG. 10.

54. Each of these figures being of an oblique shape and having no one of its angles a right angle is called an **oblique parallelogram.**

Since a rhombus has all its sides equal like a square, if a square picture frame be taken and wrested out of the perpen-

dicular, it will form a rhombus. Similarly, the frame of a school slate (which is generally rectangular or oblong) wrested out of its perpendicular will form a rhomboid.

55. To find the area of a rhombus or rhomboid.

Let *ABCD* be the oblique parallelogram whose area is required. *BC* is called the base, *i.e.* the side on which it appears to stand, *AD* is equal to *BC*, and *AE* and *DF* are called the perpendiculars upon *BC*.

Observe that the perpendiculars *AE* and *DF* form sides of a rectangular parallelogram *AEFD*. It can be proved that the area of the oblique parallelogram *ABCD* is equal to that of the rectangle *AEDF*; for if the triangle *ABE* be cut off from the oblique parallelogram *ABCD*, and placed so that *AB* may be on *DC* it will form the rectangle *AEFD*, thereby showing that the area of the oblique parallelogram *ABCD* is exactly equal to that of the rectangle *AEFD*. Now the area of the rectangle *AEFD* is found by multiplying *AD* by *AE*; therefore *AD* multiplied by *AE* also gives the area of the oblique parallelogram *ABCD* and by the definition of an oblique parallelogram the side *AD* is equal to *BC* the base. Hence, **the area of a rhombus or rhomboid is found by multiplying its base by its perpendicular height.**

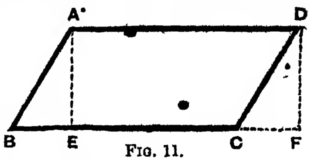


FIG. 11.

Example.—In the rhombus *ABCD*, if the base *BC* be 2 feet 1 inch and the perpendicular *AE* 1 foot 3 inches, what is the area?

2 feet 1 inch = 25 inches; 1 foot 3 inches = 15 inches.

∴ Area of rhombus *ABCD* = (25×15) square inches
= 375 square inches = 2 square feet 87 square inches.

NOTE.—As *AE* is not only the height of the rectangle *AEFD*, but also that of the rhombus *ABCD*, **the general rule for finding the area of any parallelogram—square, rectangle, rhombus or rhomboid—is to multiply the base (length) by the perpendicular height (breadth).**

EXERCISE 37.

1. Express in square yards the area of a rhombus whose base is 50 feet and height 40 feet.
2. What is the area of a rhomboid whose length is 4 feet 6 inches and perpendicular breadth 3 feet 10 inches?
3. Find the area of an oblique parallelogram whose length is $28\frac{1}{2}$ feet and perpendicular breadth $26\frac{1}{4}$ feet.

4. Find the acreage of a field in the shape of a rhomboid whose length is 242 yards and perpendicular breadth 70 yards.

5. Find the acreage of a field in the shape of a rhombus whose side measures 35 chains and whose perpendicular breadth is 150 links.

6. What is the area of a field in the shape of a rhomboid whose side measures 144 poles 2 yards and whose perpendicular breadth is 58 poles 4 yards?

7. Find the area of a field in the shape of a rhombus whose side measures 17 chains and whose perpendicular breadth is 7 chains.

8. What will it cost to cover a room in the shape of a rhomboid whose length is 15 feet and perpendicular breadth 12 feet, with carpet 2 feet wide at 3s. 6d. per yard?

56. Since the area of a rhombus or rhomboid is found by multiplying its base by its perpendicular height, it follows that if the area be known and also the base (or the perpendicular height), the perpendicular height (or the base) can be at once found. This can be expressed briefly thus:—

(1) **Area of an oblique parallelogram** $= \dfrac{\text{base}}{\text{perpendicular height}}$

(2) **Base of an oblique parallelogram** $= \dfrac{\text{area}}{\text{perpendicular height}}$

(3) **Perpendicular height of an oblique parallelogram** $= \dfrac{\text{area}}{\text{base}}$

Example.—The area of a rhomboid is 320 square yards and the base is 64 feet. Find the height.

320 square yards = 2880 square feet.

$\therefore$ Height = (2880 ÷ 64) feet = 45 feet = 15 yards.

EXERCISE 38.

1. The area of a rhombus is 49 square feet 64 square inches and the base is 7 feet 5 inches. Find the perpendicular height.

2. The area of a rhomboid is 117 square yards 5 square feet and the perpendicular height is 11 yards 1 foot. Find the length of the base.

3. The area of a rhomboid is 63 square yards 5 square feet and the base is 8 yards 2 feet. Find the perpendicular breadth.

4. The area of a rhombus is 53 square feet 18 square inches and the perpendicular height is 4 feet 3 inches. Find the length of the base.

5. The area of a field in the shape of a rhomboid is 6 acres 1 rood 16 perches 16 square yards and the perpendicular breadth is 125 yards. Find the length of the base.

6. The area of a field in the shape of a rhombus is 21 acres 2 roods 16 perches and the length of the base is 27 chains. Find the perpendicular breadth in chains.

CHAPTER IV.

TRIANGLES.

57. Triangles have three sides and three angles. A triangle which has all its sides equal is called an **equilateral** triangle (A).

58. A triangle which has only two sides equal is called an **isosceles** triangle (B).

59. A triangle which has all its sides unequal is called a **scalene** triangle (C).

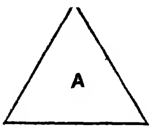


Fig. 12.

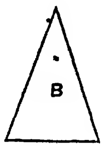


Fig. 13.

Fig. 14.

60. The **base** of a triangle is the side upon which it is supposed to stand.

61. The **perpendicular height** (or **altitude**) is the perpendicular drawn from the angular point opposite the base to the base.

62. **To find the area of any triangle having given the base and perpendicular height.**

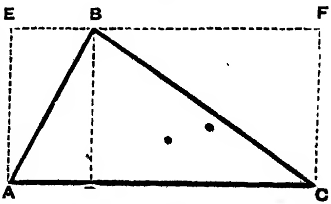


Fig. 15.

Let *ABC* be a triangle, and *AECF* be a parallelogram on the same base *AC*, and having the same height *BD*. It is evident that the triangle *ABD* is exactly half the parallelogram *AEBD*, and the triangle *DBC* half the parallelogram *DBFC*; therefore the whole triangle *ABC* is equal to half the parallelogram *AEFC*. It is plain then, that if we wish to find the area of the triangle *ABC*, we have but to find the area of the parallelogram of which it is half, and divide by two. Hence **the area of a triangle is found by multiplying the base by the perpendicular height, and taking one half the product.**

Example.—In the triangle ABC the base AC is 12 ft. 6 in. and the perpendicular height BD is 8 ft. 10 in. What is the area?

Here 12 ft. 6 in. $= 12\frac{1}{2}$ ft.; and 8 ft. 10 in. $= 8\frac{5}{6}$ ft.

$$\therefore \text{ Area of triangle } ABC = \frac{12\frac{1}{2} \times 8\frac{5}{6}}{2} \text{ sq. ft.}$$
$$= 55\tfrac{5}{24} \text{ sq. ft.}$$
$$= 55 \text{ sq. ft. } 30 \text{ sq. in.}$$

EXERCISE 39.

Find the area of a triangle, its base and perpendicular height being respectively :—

1. 7 ft. and 120 ft.
2. 484 yds. and 204 yds.
3. 2 ft. 8 in. and 1 ft. 6 in.
4. 18 ft. 4 in. and 10 ft. 3 in.
5. 3 ft. 6 in. and 8 in.
6. 25 yds. and 8 ft.
7. 400 links and 750 links.
8. 968 links and 365 links.
9. 6 ch. 25 links and 5 ch 20 links.
10. 12 ch. 50 links and 5 ch. 24 links.
11. 14·1 yds. and 9·2 yds.
12. $8\frac{3}{4}$ ft. and $7\frac{1}{2}$ feet.

EXERCISE 40.

1. A picture hangs from a single support by a cord passed through two rings in its upper edge 10 inches apart and tied together. How much wall does the cord surround if the nail be 8 inches from the picture?

2. The base of a triangular plot of ground measures 84 chains and the perpendicular upon it measured from the opposite corner 45 chains. Find its acreage.

3. What is the area of a triangular board of which the base measures 3 feet 11 inches and the perpendicular height 1 foot 3 inches?

4. Find the rent of a triangular field whose base is 4 chains 75 links and perpendicular height 25 chains at £4 per acre.

5. What will be the cost of flooring a triangular space, one side of which measures 21 feet and 15 feet being the distance of that side from the opposite one, at $8\frac{1}{2}$d. per square foot?

6. Find the cost of covering a triangular court with tiles 4 inches square; the base of the court is 12 yards and the perpendicular on it from the opposite corner is 7 yards; the tiles cost 1s. 6d. per dozen and the labour 2s. 6d. per square yard.

63. If the area of a triangle be known and also the base (or the perpendicular height), the perpendicular height (or the base) can be readily found.

Example.—The area of a triangular field is 19 acres 3 roods 8 perches and its altitude, *i.e.* its perpendicular height, is 18 chains. Find the length of the base in chains.

Here area = 19 ac. 3 ro. 8 per. = 19·8 acres.
= 1980000 square links.
And altitude = 18 chains = 1800 links.

$$\text{Since } \frac{\text{base} \times \text{altitude}}{2} = \text{area};$$

$$\frac{\text{base} \times 1800}{2} = 1980000 \text{ sq. links.}$$

$$\therefore \text{Base} \times 1800 = 3960000 \text{ sq. links.}$$

$$\therefore \text{Base} = \frac{3960000}{1800} \text{ links.}$$

$$= 2200 \text{ links.}$$

$$= 22 \text{ chains.}$$

64. From this we learn that **dividing double the area of a triangle by the perpendicular height (or the base) gives the base (or the perpendicular height)** as may be required.

EXERCISE 41.

1. The area of a triangular enclosure is 420 square feet and its perpendicular height is 15 feet. Find the length of the base.

2. The area of a triangular space is 30 square feet 88 square inches and its base is 9 feet 8 inches. Find the altitude.

3. The area of a triangular field is 5 acres 1 rood 15 perches and the base measures 45 poles. What is the altitude in poles?

4. A surveyor having lost his field book, and requiring the base of a triangular field remembered that the area was 6 acres 2 roods 8 perches and a perpendicular from one angle to the base 524 links. How much was the base in chains?

5. A triangular board contains 352¼ square inches and the base is 3 feet 11 inches. What is the perpendicular height?

6. A triangular field contains 5 acres 3 roods 5·12 perches, and its perpendicular distance measures 826 links. Find the length of the base in yards.

65. To find the area of a triangle having given the three sides.

The method of determining the area by means of the three sides is not generally used in practical work by land surveyors; the proof of the rule is too complex to be given here. It will be sufficient if we simply state the rule—

1. Add the three sides together.

2. From half this sum subtract each side separately.

3. Multiply the half sum and the three remainders continually together.

4. Extract the square root of the product: this last result will be the area of the triangle.

Example.—Find the area of a triangular plot of ground of which the sides are 26, 35 and 51 feet.

Sum of the three sides	$=(26+35+51)$ feet $=112$ feet.
Half the sum	$=112$ feet $\div 2$ $=56$ feet.
First remainder	$=(56-26)$ feet $=30$ feet.
Second remainder	$=(56-35)$ feet $=21$ feet.
Third remainder	$=(56-51)$ feet $=5$ feet.

Then $(56\times 30\times 21\times 5)$ sq. feet $=176400$ sq. feet.
And $\sqrt{176400}=420$.
$\therefore$ Area of the plot $=420$ square feet.

EXERCISE 42.

Find the area of the triangles having the following sides :—

1. 16 ft., 63 ft., 65 ft.
2. 39 ft., 42 ft., 45 ft.
3. 26 ft., 51 ft., 55 ft.
4. 20 yds., 21 yds., 29 yds.
5. 25 yds., 113 yds., 132 yds.
6. 20 yds., 493 yds., 507 yds.
7. 25 ch., 20 ch., 15 ch.
8. 29 ch., 52 ch., 69 ch.
9. 7 ch., 12 ch., 17 ch.
10. 45 ch., 40 ch., 13 ch.
11. 456 links, 512 links, 600 links.
12. 12 ch. 45 links, 15 ch. 70 links 16 ch. 25 links.

N.B.—*The last six results are to be given in acres, etc.*

EXERCISE 43.

1. The three sides of a triangular piece of wood measure 15, 28 and 41 inches respectively. What space does it cover?

2. The sides of a triangular portion of a field are 25, 113 and 132 yards respectively; and this portion is an eleventh part of the whole field. Find the acreage of the field.

3. A triangular field whose sides are 350, 440 and 750 yards is let for £26 5s. a year. What is that per acre?

4. Find the rent of a triangular field whose three sides measure 2470, 2250 and 2120 links at £2 10s. per acre

5. Required the expense of reaping the corn in a triangular field whose sides are respectively 150, 200 and 250 yards long at 10s. per acre.

6. What is the rent of a triangular field whose sides are respectively $7\frac{1}{2}$, $8\frac{1}{4}$ and $10\frac{3}{4}$ chains long at £2 5s. per acre?

7. How many tiles a foot square will be required to pave a triangular court whose sides are 30, 40 and 50 feet respectively?

8. The sides of a triangular field are 12, 18 and 24 chains long respectively. What is its acreage?

66. Sometimes triangles are named from their angles; thus a **right-angled** triangle is one that has a right angle (A).

67. An **obtuse-angled** triangle is one that has an obtuse angle (B).
68. An **acute-angled** triangle is one that has three acute angles (C).

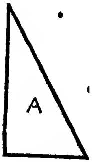


FIG. 16

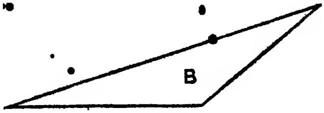


FIG. 17

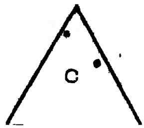


FIG. 18

69. In right-angled triangles the three sides have been given special names according to their position with regard to the right angle.

Choosing either of the two sides forming the right-angle as the base, the other is the perpendicular, while the third or longest side is called the hypotenuse.

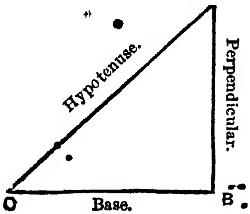


FIG 19.

Thus in the triangle ABC, the side AC opposite the right-angle is the **hypotenuse,** the side BC being called the **base** and the side AB the **perpendicular.** The sides AB and BC which include the right-angle are sometimes termed the **legs** of the triangle.

70. The sides of every right-angled triangle have this property, that a square described on the hypotenuse is equal to sum of the squares described on the other two sides. The diagram annexed shows the truth of this statement.

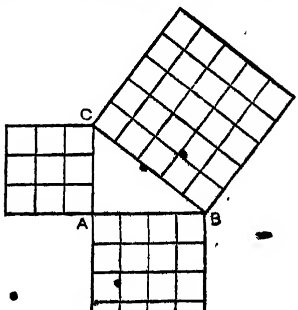


FIG. 20.

Let ABC be a right-angled triangle BC being the hypotenuse and AB and AC the two sides containing the right-angle BAC.

Assume that AB is 4 inches long and AC 3 inches long. Now if squares be constructed on AB and AC, made up of pieces of cardboard each an inch square, it will be found that 16 of these smaller squares will be required for the square on AB and 9 for the square on AC, or 25 smaller squares on both sides together. If these smaller squares be now

removed, and taken to form a square on BC, it will be found that the 25 smaller squares are required to construct it.

71. **Hence to find the length of the hypotenuse of a right-angled triangle when the lengths of the sides which include the right-angle are given, add the squares of the sides and extract the square root of their sum.**

Example. The base of a right-angled triangle is 20 yards long, and its perpendicular 15 yards. Find the length of the hypotenuse.

Here the square described on the base $=20^2=400$ sq. yards.
And the square described on the perpendicular $=15^2=225$ sq. yards.
$\therefore$ The square on the hypotenuse $=(400+225)$ sq. yards $=625$ sq. yards.
$\therefore$ Length of the hypotenuse $=\sqrt{625}$ yards $=25$ yards.

EXERCISE 44.

Find the hypotenuse of a right-angled triangle of which the base and perpendicular height are respectively :—

1. 12 in. and 16 in.
2. 27 ft. and 36 ft.
3. 72 ft. and 65 ft.
4. 357 ft. and 476 ft.
5. 84 ch. and 63 ch.
6. 1164 yds. and 873 yds.
7. 4 ft. and 1 ft. 8 in.
8. 1 ft. 9 in. and 2 ft. 4 in.
9. 2 ft. 9 in. and 3 ft. 8 in.
10. 16 yds. and 6 yds. 2 ft.
11. 3 yds. 1 ft. and C ft. 3 in.
12. 67·5 chains and 5·2 chains.

EXERCISE 45.

1. The top of a ladder touches a wall 4 yards above the ground when its foot is placed 3 feet 2 inches from the bottom of the wall. How long is the ladder?

2. One end of a cord is tied to the top of a tree 24 feet high, and the other end to a stake placed 18 feet from the tree. What is the length of the cord?

3. A tower 36 feet high stands on a bank of a river 27 feet broad. What length of cord will just reach from the top of the tower to the opposite bank of the river?

4. A house is 48 feet high and the street in front 14 feet wide. Find the length of a ladder which will just reach the top of the house from the opposite side of the street.

5. A straight pole was broken 9 feet from the bottom and fell so that the end struck 12 feet from the bottom of the pole. Find the whole length of the pole.

6. The town C lies 45 miles east from B and 60 miles distant from A which lies north from C. What is the distance from B to A?

7. Barcelona is 188 miles north-east of Valencia and 570 miles north-west of Tunis. Find the distance of Tunis from Barcelona.

8. John starts on Monday and walks 10 miles a day due south, George starts from the same place on Tuesday and walks 20 miles a day due west. How far apart will they be on Wednesday night?

9. Two bicyclists sent out from a certain point, one going south-east at the rate of 10 miles an hour and the other south-west at the rate of 12 miles an hour. How far apart will they be at the end of 4 hours?

10. Two straight roads cross at right angles; two men start at the same time from the point where the roads meet, one man going along one road at the rate of 4 miles an hour and the other man walking along the other road at the rate of 3 miles an hour. How far apart will the men be in 10 minutes after starting?

72. From the diagram (§ 70) it will be seen that when the hypotenuse and either of the two other sides are given, the length of the third side can be readily ascertained. Thus, in the diagram the number of small squares (16) on the side *AB* is equal to the difference between the number of small squares on *BC* (25) and *AC* (9); likewise the number of small squares on *AC* (9), is equal to the difference between the number of small squares on *BC* (25) and *AB* (16); therefore

73. To find the remaining side of a triangle when the hypotenuse and either of the other sides are given, from the square of the hypotenuse subtract the square of the given side and extract the square root of the remainder.

Example.—The hypotenuse of a right-angled triangle is 65 feet and one of its sides is 52 feet. Find the length of the other side.

Here the square of the hypotenuse $= 65^2 = 4225$ sq. feet.
And the square of the given side $= 52^2 = 2704$ sq. feet.
$\therefore$ The square of the side required $= (4225 - 2704)$ sq. ft. $= 1521$ sq. ft.
$\therefore$ Length of the side required $= \sqrt{1521}$ feet $= 39$ feet.

EXERCISE 46.

Find one side of a right-angled triangle, of which the hypotenuse and the other side are respectively:—

1. 65 ft. and 25 ft.
2. 95 ft. and 57 ft.
3. 70 ft. and 42 ft.
4. 170 ft. and 154 ft.
5. 105 ch. and 84 ch.
6. 500 links and 400 links.
7. 6 ft. 1 in. and 4 ft. 7 in.
8. 2 ft. 10 in. and 1 ft. 4 in.
9. 2 ft. 1 in. and 1 ft. 3 in.
10. $17\frac{1}{2}$ yds. and 14 yds.
11. 6·5 ft. and 5·2 ft.
12. 3 yds. 1 ft. 3 in. and 3 yds. 1 ft.

EXERCISE 47.

1. A ladder 45 feet long reaches a window 36 feet from the ground. Find the distance of the foot of the ladder from the side of the house.

2. A ladder when standing upright against a wall reaches $12\frac{1}{2}$ feet up the wall. How far must the bottom of the ladder be pulled out from the wall to lower the top six inches?

3. A line 45 feet long will reach from the top of a tower 27 feet high, standing on the bank of a river, to the opposite bank. Required the breadth of the river.

4. A ladder when standing upright against a wall reaches 65 feet up the wall. If its foot be drawn out 16 feet, how far will the top descend?

5. A river 30 feet broad flows round the base of a tower. If a line 50 feet long will reach from the opposite bank to the top of the tower, what is its height?

6. *A* and *B* are 75 yards apart. If by walking north and west respectively they meet, and to do this *A* has to walk 60 yards, how many does *B* walk?

7. A ladder 40 feet long is placed so as to reach a window 24 feet high on one side of a street, and on turning the ladder over to the other side of the street it reaches a window 32 feet high. Find the breadth of the street.

8. In a gale a flagstaff 20 yards high, standing on level ground, snaps 28·8 feet from the bottom, and the upper portion not being wholly broken off, the top touches the ground. How far is the point of contact with the ground from the bottom of the staff?

74. To find the area of a right-angled triangle when the length of the hypotenuse and base (or perpendicular height) only are given, it will be necessary to find the other side (§ 73) before applying the rule (§ 62).

EXERCISE 48.

Find the area of a right-angled triangle, of which the hypotenuse and one side are respectively:—

1. 400 ft. and 240 ft.
2. 12 ft. 6 in. and 12 ft.
3. 5 ft. 1 in. and 11 in.
4. 464 links and 368·4 links.
5. 13 ch. 96 links and 13 ch. 67 links.
6. 290 yds. and 200 yds.

N.B.—*The last three results are to be given in acres, etc.*

75. To find the area of an isosceles triangle the length of the equal sides being given.

Let ABC be an isosceles triangle having equal sides AB and AC. If a perpendicular AD be drawn from the vertex A of the triangle, it bisects the base BC, and the triangle ABC is divided into two right-angled triangles. Now, since AD bisects BC, BD must be half of BC, and we can find the length of the perpendicular AD by the method given in § 73, whence the area of the triangle will be found as before by multiplying the base by the perpendicular height.

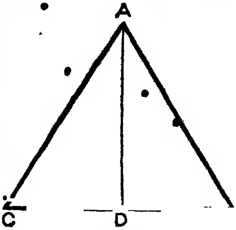


FIG. 21.

In like manner the area of an equilateral triangle may be found.

Example.—Find the area of an isosceles triangle ABC, when the base BC is 24 feet and each of the equal sides AB and AC 20 feet.

Here we must first find AD, the perpendicular height of the triangle.

Since AD bisects BC, BD is half of 24 feet = 12 feet · and since ADB is a right-angled triangle

$$AD = \sqrt{20^2 - 12^2} \text{ feet} = \sqrt{256} \text{ feet} = 16 \text{ feet}.$$

$$\therefore \text{Area of triangle } ABC = \frac{24 \times 16}{2} = 192 \text{ square feet.}$$

EXERCISE 49.

Find the area of an isosceles triangle, having the base and equal sides respectively :—

1. 10 inches and 12 inches.
2. 136 feet and 85 feet.
3. 48 yards and 40 yards.
4. 2 ft. 6 in. and 8 ft. 3 in.
5. 6 feet and 7 ft. 1 in.
6. 14 feet and 13 ft. 3 in.

AREA OF PARALLELOGRAMS FROM DIAGONAL.

76. A straight line joining the opposite angles of a rectangle (or a square) is called a **diagonal.** Thus AC is the diagonal of the rectangle $ABCD$, and from the figure it will be seen that (1) the diagonal AC divides the rectangle into two right-angled triangles; and (2) the diagonal AC is the common hypotenuse of the right-angled triangles ABC and ADC.

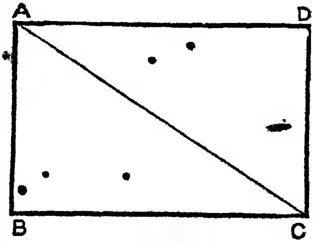


FIG. 22.

77. The diagonal of a square is the hypotenuse of a right-angled triangle, whose base and perpendicular height are two of the equal sides of the square (§ 69).

78. To find the area of a square when the length of a diagonal is given.

In the triangle ABC it is evident that the square on AC equals the sum of the squares on AB and BC, but being sides of a square AB equals BC; therefore the square on AC equals twice the square on AB. But AB squared equals the area of the square; therefore the square on the diagonal of a square is twice the area of the square; hence **to find the area of a square from the diagonal square the diagonal and take one-half of this result.**

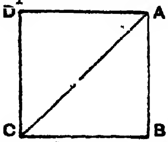


FIG. 23.

EXERCISE 50.

Find the area of the squares whose diagonals are respectively :—

1. 110 yards.	3. 9 ft. 9 in.	5. 16 chains.
2. 1112 yards.	4. 1000 links.	6. 7 chains 15 links.

N.B.—*The last three results are to be given in acres, etc.*

79. The diagonal of a rectangle is the hypotenuse of a right-angled triangle, of which the sides are the length and breadth of the rectangle.

80. To find the area of a rectangle from the diagonal, the length of one of the sides must be also given; the remaining side of the triangle, which is the other side of the rectangle, can then be ascertained (§ 73) and, lastly, the area of the rectangle (§ 33).

EXERCISE 51.

1. Find the area of a rectangular lawn of which the diagonal is 100 yards and the breadth 60 yards.

2. Find the area of the rectangle having a diagonal 4 feet 5 inches and a side 2 feet 4 inches.

3. Find the acreage of a field whose width is 111 yards, the distance from corner to corner being 185 yards.

4. The diagonal of a square courtyard is 90 feet. Find the cost of gravelling the court at the rate of 1s. for every nine yards.

5. The diagonal of a square field is 7 chains 15 links. Find the rent of it at £2 5s. per acre.

6. A footpath goes along the adjacent sides of a rectangular field 196 yards long and 147 yards broad. What distance would be saved if the path were straight across the field from corner to corner?

7. A rectangular field is 56 yards long, and the distance from corner to corner is 65 yards. How many yards of fencing will be required to enclose it?

8. Two travellers, A and B, arrive at the corner of a rectangular lake. A goes to the opposite corner in a boat, a distance of $1\frac{1}{4}$ miles; B walks along the shore of the lake, intending to rejoin him, and has to go a mile before he turns the end of the lake. How much farther has B to go before he rejoins A?

81. To find the area of a rhombus having given the two diagonals.

Let $ABCD$ be a rhombus, then AC and BD are the two diagonals. The diagonals of a rhombus bisect each other at right angles, and each diagonal divides the rhombus into two equal parts; thus diagonal BD divides $ABCD$ into two triangles BAD and BCD. Now the area of triangle $BAD = \frac{1}{2}(BD \times AE)$ and the area of triangle $BCD = \frac{1}{2}(BD \times EC)$; therefore the area of the whole figure $ABCD = \frac{1}{2}BD \times (AE + EC)$, that is the area of $ABCD = \frac{1}{2}(BD \times AC)$; hence **the area of a rhombus may be found by multiplying the two diagonals together and taking one-half of this result.**

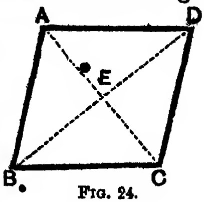


FIG. 24.

Example.—Find the area of a rhombus whose diagonals are respectively 750 and 600 links.

$$\text{Here area} = \frac{750 \times 600}{2} = 225000 \text{ square links.}$$
$$= 2 \text{ acres } 1 \text{ rood.}$$

It follows that if **the area of a rhombus be known and also the length of one of its diagonals, the other diagonal will equal twice the given area divided by the given diagonal.**

EXERCISE 52.

Find the area of a rhombus whose diagonals are respectively :—

1. 30 feet and 50 feet.
2. 421 feet and 248 feet.
3. 96 yards and 100 yards.
4. 88 yards and 110 yards.
5. 64 yards and 110 yards.
6. 1980 links and 1125 links.

N.B.—*The last three results are to be given in acres, etc.*

EXERCISE 53.

1. A four-sided field $ABCD$ is measured with a chain from each corner to the opposite corner. AC is found to be 10 chains 11 links and BD 11 chains 10 links. AC and BD are at right angles to one another. What does the field measure in acres, roods and perches?

2. Find the length of the side of a square whose area is equal to the area of a rhombus whose diagonals are 52 feet and 416 feet.

3. A floor 90 feet by 36 feet is paved with tiles each of a rhomboidal shape with diagonals 9 inches and $7\frac{1}{2}$ inches respectively. How many tiles will be required?

4. The area of a rhombus is 990 square feet and one of its diagonals measures 55 feet. Find the length of the other diagonal.

5. The area of a field in the shape of a rhombus is $130\frac{1}{2}$ acres, and one of its diagonals measures 45 chains. What is the length of the other diagonal?

6. The area of a field in the shape of a rhombus is o acres, 1 rood 8 perches and the longer diagonal measures 1325 links. How many links does the shorter diagonal measure?

EXERCISE 54.

Revision Examination.—Triangles and Parallelograms.

1. Find the area of an equilateral triangle whose sides measure 25 feet.

2. What is the height of an equilateral triangle whose sides measure 10 feet?

3. The sides of a right-angled plot of ground measure 1·5 and 2 chains respectively. Find the perpendicular distance in yards from the right-angle to the opposite side.

4. The legs of a right-angled triangle are 36 and 48 feet. Find the perpendicular distance from the right-angle to the hypotenuse.

5. Find the area of an equilateral triangle whose perimeter is 114 feet.

6. A triangle of which the three sides are 3161, 3111 and 560 inches respectively is equal in area to an isosceles triangle of which the altitude is 1220 inches. Find the length of the base of the latter triangle.

7. What is the length of the side of a square field whose area is equal to that of a triangular field with sides of 182, 168 and 70 yards respectively?

8. Find the difference between the area of a triangle whose sides are 16, 63 and 65 feet long, and the area of a square whose perimeter is the same as that of the triangle.

9. From an isosceles triangle whose base and equal sides are respectively 9 feet and 6 feet, is cut a square 3 square feet in area. Find (to two places of decimals) the length of a side of a square whose area is that of the remainder of the triangle.

10. *ABCD* is a parallelogram whose base *DC* is 16 yards 2 feet 3 inches and height 14 yards 2 feet 8 inches; *AB* is produced to any point *E*. Find the area of the triangle *DCE*.

11. One side of a right-angled triangle is 119 feet. Find the other side, if its area be the same as that of a triangle whose sides are 20, 493 and 507 feet respectively.

12. *ABCD* is a field of which two opposite angles at *B* and *D* are right angles. If *AB* be 300 yards, *BC* 400 yards, *CD* 100 yards, find the area in square yards.

CHAPTER V.

THE TRAPEZOID AND TRAPEZIUM.

82. We have dealt with four quadrilateral figures, viz., the rectangle, the square, the rhombus, and the rhomboid; there remain two others, the trapezoid and the trapezium.

83. A **trapezoid** is a four-sided figure which has two of its sides parallel; as (A) or (B).

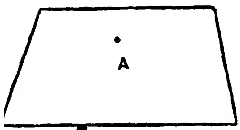


Fig. 25.

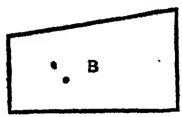


Fig. 26.

84. **To find the area of a trapezoid when the lengths of two parallel sides and the perpendicular distance between them are given.**

Let *ABCD* be a trapezoid, of which *AB* and *DC* are parallel sides, and *CE* the perpendicular distance between them. The trapezoid may be divided into two triangles by the diagonal

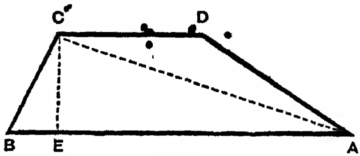


Fig. 27.

AC. Now the area of triangle $DAC=\frac{1}{2}(DC\times CE)$, and the area of triangle $ACB=\frac{1}{2}(AB\times CE)$; therefore the area of the whole figure *ABCD* must be $(AB+DC)\times\frac{1}{2}CE$; hence **to find the area of a trapezoid, multiply the sum of the parallel sides by half the perpendicular distance between them.**

Example.—The two parallel sides of a trapezoid are $5\frac{1}{3}$ yards and 7 yards, and the perpendicular distance between them is 8 yards. Find the area.

Here sum of parallel sides $=(5\frac{1}{3}+7)$ yards $=12\frac{1}{3}$ yards.
Half the perpendicular distance $=$ 4 yards.
$\therefore$ Area $=(12\frac{1}{3}\times 4)$ sq. yards $=49\frac{1}{3}$ sq. yds. $=49$ sq. yds. 3 sq. ft.

EXERCISE 55.

Express each answer in the highest denomination.

Find the area of the trapezoids having the following dimensions :—

1. Parallel sides 34 ft. and 14 ft. ; perpendicular distance $127\frac{1}{2}$ ft.
2. Parallel sides 234 in. and 104 in. ; perpendicular distance 92 in.
3. Parallel sides 18 ft. 6 in. and 20 ft. 8 in. ; perpendicular distance 10 ft. 4 in.
4. Parallel sides 421 yds. and 317 yds. ; perpendicular distance 264 yds.
5. Parallel sides 37 chains and 13 chains ; perpendicular distance 25 chains.
6. Parallel sides 80 links and 60 links ; perpendicular distance 540 links.

EXERCISE 56.

1. Find the area in square chains of a quadrilateral field of which two parallel opposite sides are respectively 750 links and 1225 links, and the perpendicular distance between these sides is 1540 links.
2. A field is in the form of a trapezoid ; its parallel sides are respectively 10 chains 30 links and 7 chains 70 links ; the distance between them is 7 chains 50 links. Find the acreage.
3. The sum of the parallel sides of a trapezoidal field being 11 chains and the perpendicular distance between them 150 yards, find the acreage.
4. Find the length of a side of a square whose area is equal to that of a trapezoid whose parallel sides are $5\frac{1}{4}$ inches and 7 inches respectively, and the perpendicular distance between them 50 inches.
5. What is the rent at £1 11s. per acre of a field in the form of a trapezoid whose parallel sides are 6340 yards and 4380 yards, and the perpendicular distance between them 121 yards?
6. Find the cost of boarding a floor in the form of a trapezoid whose parallel sides are 16 feet 8 inches and 14 feet 10 inches, and the perpendicular distance between them 8 feet 4 inches at $3\frac{1}{2}$d. per square foot.

85. A trapezium is a four-sided figure which has none of its sides parallel.

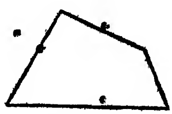


FIG. 28

86. To find the area of a trapezium.

Let $ABCD$ be a trapezium, and let BD be one of its diagonals, and AE and FC perpendiculars on the diagonal from the opposite corners. $ABCD$ is divided into two triangles ABD and BCD by BD. Now area of triangle $ABD = \frac{1}{2}\ (BD \times AE)$ and the area of triangle $CBD = \frac{1}{2}\ (BD \times CF)$; therefore the area of the whole figure $ABCD$ must be $(AE + CF) \times \frac{1}{2}\ BD$; hence **to find the area of a trapezium multiply half the diagonal by the sum of the perpendiculars upon it.**

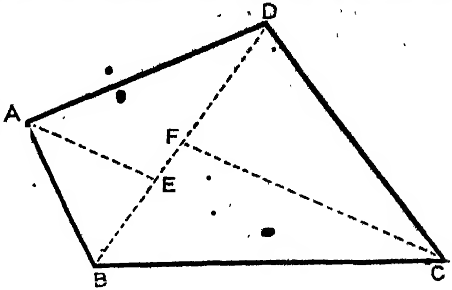


Fig. 29.

Example 1.—Find the area of the trapezium $ABCD$, given that the diagonal BD is 27 yards and the perpendiculars upon it from A and C 21 yards and 14 yards respectively.

Here sum of the perpendiculars $= (21 + 14$ yards$) = 35$ yards.
Half the diagonal $= 13\frac{1}{2}$ yards.
$\therefore$ Area $= (13\frac{1}{2} \times 35)$ sq. yards. $= 472\frac{1}{2}$ sq. yards.
$= 472$ sq. yds. 4 sq. ft. 72 sq. ins.

Example 2.—Find the acreage of a trapezium $ABCD$, given that the diagonal AC is 325 yards and the sides AB, BC, CD, DA are 123, 208, 146 and 231 yards respectively.

The diagonal AC divides the trapezium into two triangles, the sides of which are known.

The area of the first triangle $= 4920$ sq. yards.
The area of the second triangle $= 9240$ sq. yards.
$\therefore$ Area of trapezium $ABCD = 14160$ sq. yards.
$= 2$ ac. 3 ro. 28 per. 3 yds.

Note.—The pupil should draw a figure $ABCD$ with sides proportional to the given dimensions.

EXERCISE 57.

In every case draw a diagram.

1. Find the area of a trapezium whose diagonal is 20 yards and the two perpendiculars 4·2 and 3·8 yards respectively.

2. In surveying a four-sided field I found the diagonal measures 750 links and the perpendiculars upon it from the opposite corners 280 links and 420 links respectively. Find its area in acres, roods and perches.

3. *ABCD* is a quadrilateral field of which the diagonal *AC* is 264 yards, the perpendicular from angle *B* to the diagonal 66 yards and from angle *D* 132 yards. Find its area in acres, roods and perches.

4. Find the area of a trapezium *ABCD* of which *AB* is 28 yards, *BC* 45 yards, *CD* 51 yards, *DA* 52 yards, and the diagonal *AC* 53 yards.

5. The diagonal *AC* of a four-sided field *ABCD* is 760 links. The sides *AB*, *BC*, *CD* and *DA* are 600, 480, 320 and 540 links respectively. Find its acreage.

6. *ABCD* is a quadrilateral. *AB* measures $62\frac{1}{2}$ feet, *BC* $107\frac{1}{2}$, *CD* $127\frac{1}{2}$ and *AD* 150; and the diagonal *AC* $132\frac{1}{2}$ feet. Find the area in square yards.

7. *ABCD* is a quadrilateral. The diagonal *AC* measures $132\frac{1}{2}$ feet and the perpendiculars to it from *B* and *D* are 50·075 and 120·15 feet respectively. Find the area in square yards.

8. A diagonal of a four-sided field is 660 yards and the sum of the perpendiculars on the diagonal is 143 yards. Find the rent of the field at £3 4s. per acre.

9. A diagonal of a four-sided field is 2 chains 50 links, and the perpendiculars on it from the opposite angles 1 chain 20 links and 1 chain 80 links. Find the value of the field at 1s. 4d. per square yard.

10. A diagonal of a four-sided field is 18 chains 75 links, and the perpendiculars on it from two corners are 6 chains 54 links and 7 chains 82 links. Find its rent at 12s. per rood.

CHAPTER VI.

POLYGONS.

87. Figures which are bounded by more than four straight lines are called **polygons.***

88. When the sides and angles of a polygon are all equal the the figure is termed a *regular polygon.*

89. When the sides and angles are unequal the figure is termed an *irregular polygon.*

90. Regular polygons with five sides are called *pentagons*; those with six sides *hexagons*; those with seven sides *heptagons*; those with eight sides *octagons*; those with nine sides *nonagons*; those with ten sides *decagons.*

* The word *polygon* signifying *many cornered* is derived from the Greek *polys* many, and *gonia* an angle.

91. To find the area of a regular polygon when the side and perpendicular upon it are given.

Let $ABCDE$ be a regular polygon, and let O be a point within the figure, at equal perpendicular distance from each side. This point O is called the centre of the polygon. From O draw OF perpendicular to the middle of the side CD. From the centre draw straight lines to each angle of the figure; then the polygon is divided into as many equal triangles as the figure has sides. Now the area of triangle COD is equal to half the product of the side CD and the perpendicular OF (§ 62); therefore the area of all the triangles is equal to the area of the polygon. Thus **the area of a regular polygon is equal to half the sum of the sides multiplied by the perpendicular drawn from the centre of the polygon to the middle point of one of its sides.**

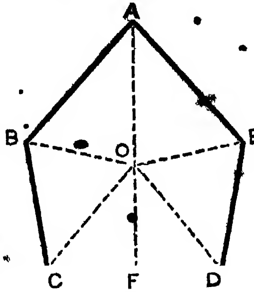


FIG. 30.

NOTE.—The sum of the sides of a regular polygon is called its *perimeter*.

Example.—In the pentagon $ABCDE$, if each side be 8 feet and the perpendicular OF 7 feet, what is the area?

$$\text{Here area of the trinagle } OCD = \frac{8 \times 7}{2} \text{ sq. feet.}$$

$$\therefore \text{ Area of the pentagon} = \frac{8 \times 7 \times 5}{2} \text{ sq. feet.}$$

$$= 140 \text{ square feet.}$$

EXERCISE 58.

Find the area of the following regular polygons, each side (and perpendicular) being respectively :—

1. Pentagon, side 3 ft. ; perpendicular 2 ft. 1 in.
2. Heptagon, side 17 ft. 6 in. ; perpendicular 18 feet.
3. Hexagon, side 30 poles ; perpendicular $26\frac{1}{2}$ poles.
4. Octagon, side 23·75 chains ; perpendicular 28·25 chains.
5. Nonagon, side 500 links ; perpendicular 686·8 links.
6. Decagon, side 60 yards ; perpendicular 92·3 yards.

N.B.—*Give the last four results in acres, roods and perches.*

92. To find the area of any polygon.

Let it be required to find the area of the polygon $ABCDE$.

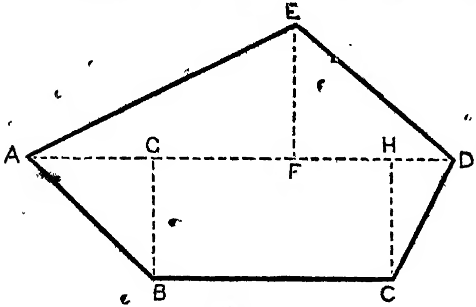


Fig. 31.

Draw the greatest diagonal AD. On this drop perpendiculars from the points B, C and E; the polygon is thus divided into three triangles ABG, CDH, AED and the rectangle $GBCH$. It is evident that the sum of the areas of these must equal the area of the whole figure; hence **to find the area of a polygon divide the figure into convenient parts and find the area of each part separately; the sum of these areas is the area of the polygon.**

The line joining the two farthest angles may be called the *base line* and when a perpendicular is drawn from an angle to this line, such a perpendicular is termed an *offset*: thus EF, GB, HC are offsets from AD. These terms are used in surveying or measuring a field. The measurements are entered by surveyors in a *Field Book*, and are taken by the chain and entered in links.

Example. In the field $ABCDE$ a base line AD—351 links—is measured; 71 links along AD, a perpendicular offset—84 links—is made to B; 205 links along AD an offset—129 links—is made to E; and 305 links along AD an offset—84 links—is made to C. Find the area of the field.

Now the field as arranged consists of three triangles and a rectangle.

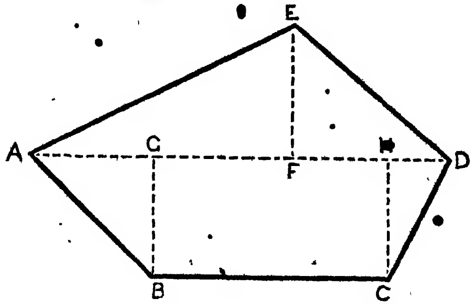
Area of triangle	$ABG=$	$\frac{71\times 84}{2}$	$= 2982$ sq. links.	
Area of rectangle	$GBCH=$	84×234	$=19656$	,,
Area of triangle	$CDH=$	$\frac{46\times 84}{2}$	$= 1932$	,,
Area of triangle	$AED=$	$\frac{351\times 129}{2}$	$=22639{\cdot}5$	,,

$\therefore$ Area of field $ABCDE = 47209{\cdot}5$ sq. links.
$= 1$ rood $35{\cdot}5$ perches.

Note.—When asked to draw a plan of a field from the measurements in the Field Book, allow about 2 inches to 100 links.

93. The Field Book is arranged in three columns. In the

middle column, commencing from the bottom, the surveyor enters the distances from the starting-station, measured on the base line of those points from which offsets are taken. In the right hand and left hand columns respectively are entered the right and left hand offsets.



In the above figure, if it be taken to represent a field, the entries would be made in the Field Book thus :—

	To D	
	351	
	305	to C 84
to E 129	205	
	71	to B 84
	From A	go east

EXERCISE 59.

Draw the plans and find the acreage of the fields whose measurements (in links) are given in the following Field Books :—

	1.	
	to B	
	313	
to E 97	255	
	148	to D 75
to C 50	98	
	From A	

	2.	
	to B	
	475	
to E 90	420	
	320	to D 160
to C 160	175	
	From A	

	3.	
	to B	
	200	
to E 80	173	
	100	to D 96
to C 100	97	
	From A	

	4.	
	to B	
	874	
	730	to E 44
to D 50	374	
	250	to C 136
	From A	

5.

	to B	
	1000	
to F 88	625	
	500	to E 66
to D 176	375	
	250	to C 121
	From A	

6.

	to B	
	960	
to F 450	720	
	640	to E 300
to D 325	520	
	480	to C 250
	From A	

7.

	to B	
	600	
to G 200	560	
to F 150	480	
	470	to E 150
to D 100	380	
	100	to C 200
	From A	

8.

	to B	
	700	
	600	to G 150
to F 350	500	
	350	to E 250
to D 250	150	
	100	to C 200
	From A	

9.

	to B	
	1278	
to E 116	1094	
	944	to D 200
to F 90	764	
	544	to C 154
to G 200	388	
to H 352	248	
	From A	

10.

	to B	
	600	to H 80
to G 140	560	
to F 150	480	
	470	to E 200
to D 100	380	
	100	to C 150
	From A	

EXERCISE 60.

Examination Tests.—Rectilineal Figures.

A.

1. How many wooden blocks $9\frac{1}{2}$ inches long and $4\frac{1}{4}$ inches wide would be required to pave a floor measuring 25 feet 4 inches by 19 feet $1\frac{1}{2}$ inches?

2. At 8s. 8d. per acre, required the expense of reaping a triangular field whose sides are 37·24, 49·35 and 39·59 chains.

3. A field is bounded by four straight lines, of which two are parallel. If the sum of the parallel sides be 1235 links, and the perpendicular distance between them 240 links, determine the area of the field in square links.

B.

4. Find the expense of covering the walls of a room 25 feet long, 23 feet $7\frac{1}{2}$ inches wide and 16 feet high with paper 1 foot 9 inches wide, at $10\frac{1}{2}$d. a yard.

5. Find the acreage of a field in the form of a trapezium whose diagonal is 1660 links and the perpendiculars on it from opposite corners are 702 and 712 links.

6. One end of a rope 52 feet long is tied to the top of a pole 48 feet high, and the other end is fastened to a peg in the ground. If the pole be vertical and the rope tight, find how far the peg is from the foot of the pole.

C.

7. Find the number of yards in a side of a square field the area of which is 11 acres 36 perches.

8. A field in the form of a trapezoid whose parallel sides are 6340 and 4380 yards and the perpendicular distance between them 121 yards, lets for £1 11s. per acre. Find the rent.

9. The houses in a street are 40 feet high and the street is 30 wide. Find the length of the ladder which will reach from the top of one of the houses to the opposite side of the street.

D.

10. A rectangular field four times as long as it is broad contains $2\frac{1}{2}$ acres. Find in yards the length and breadth of the field.

11. The height of a tower on the brink of a river is 42·426 feet and the breadth of the river is 23 yards. How many yards of cord will reach from the top of the tower to the opposite bank of the river?

12. The length of the diagonal of a four-sided field is 54 feet and the lengths of the perpendiculars upon the diagonal from the opposite corners are 23 feet 9 inches and 18 feet 3 inches. How many square yards are there in the field?

E.

13. Calculate to the tenth of an inch the side of a square field which is an acre in area.

14. Two ships sail from the same port; one sails due west, at the rate of 10 miles an hour and the other due north at the rate of $8\frac{1}{2}$ miles per hour. How far apart are they at the end of 4 hours?

15. Find the expense of covering with lead at a farthing per square inch the inside of a cistern, open at the top, of length 10 feet, width 6 feet and depth 4 feet.

F.

16. How many yards of wall paper $22\frac{1}{2}$ inches wide will be required for a room 20 feet long, 16 feet broad and 10 feet high? Sixty square feet are to be deducted for window and door space.

17. A field in the form of a parallelogram contains 3 acres. Find its length if the base be 240 yards.

18. The diagonal of a square courtyard is 30 yards. Find the cost of gravelling the courtyard at $10\frac{1}{2}$d. for 9 yards.

G.

19. What will be the cost of painting a room 22 feet long, 18 feet wide and 11 feet high at 2s. 3d. per square yard, allowance being made for two windows each 6 feet by 4 feet and a fireplace 5 feet by 6 feet?

20. A path 8 feet wide surrounding a rectangular court 60 feet long and 36 feet wide is to be paved with tiles 9 inches long and 4 inches wide. How many will be required?

21. Find the area of an isosceles triangle whose base is 3 feet and each of whose equal sides is 5 feet.

H.

22. How many rectangles each measuring 10 feet $9\frac{1}{3}$ inches by 5 feet $2\frac{1}{2}$ inches are there in one whose area is 12125 square feet?

23. What is the length of the diagonal of a square whose area is 7 square inches?

24. The diagonal of a trapezium is 50·08 feet and the perpendiculars upon it from the two opposite angles are 10·12 feet and 8·4 feet. Find the area.

CHAPTER VII.

THE CIRCLE.

94. A **circle** is a plane figure bounded by a curved line called the circumference, and is such that all straight lines drawn from a point within the figure, called the centre, to the circumference are equal.

Thus *ABCD* is the **circumference** and *O* the **centre**.

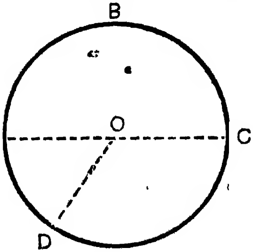


FIG. 32.

95. Each of the straight lines drawn from the centre to the circumference is called a **radius** (plural *radii*). *OA* is a radius; *OC* and *OD* are also radii. All the radii of a circle are equal.

96. A straight line drawn through the centre and terminated both ways by the circumference is called a **diameter.** *AC* is a diameter. It follows that the diameter of a circle is twice the length of the radius.

97. Every diameter of a circle bisects it, that is, divides it into two equal parts, each of which is called **a semicircle.**

98. In all circles the circumference is about $3\frac{1}{7}$ (or more accurately 3·1416) times the length of the diameter; hence

99. **To find the circumference of a circle, the diameter being given, multiply the diameter by $\frac{22}{7}$.**

Example.—Find the circumference of a circle the diameter of which is 36 feet 9 inches.

$$\text{Since } 36 \text{ ft. } 9 \text{ in.} = 36\tfrac{3}{4} \text{ ft.} = \frac{147}{4} \text{ ft.};$$

$$\therefore \text{Circumference} = \frac{147}{4} \times \frac{22}{7} = \frac{21 \times 11}{2} \text{ feet} = 115 \text{ feet } 6 \text{ inches.}$$

EXERCISE 61.

Find the circumferences of the circles whose diameters are :—

1. 63 feet.
2. 42 yards.
3. 98 yards.
4. 6·32 yards.
5. 18·8 feet.
6. 135·6 inches.
7. 3 feet 9 inches.
8. 11 feet 1 inch.
9. 22 feet 9 inches.

EXERCISE 62.

1. Find the circumference of a pillar whose diameter is $10\frac{1}{2}$ feet.
2. If the diameter of a well be 7 feet 6 inches, what is its circumference?
3. Assuming the radius of a wheel to be $6\frac{1}{4}$ inches, how many inches round is the wheel?
4. A circular table measures $67\frac{1}{2}$ inches across its widest part. What is its circumference?
5. The diameter of a circular plantation is 824 links. How many yards of paling would enclose it?
6. The diameter of the earth being 7912 miles, what is its circumference?
7. The circumference of a wheel being 4 feet 7 inches, how many yards will it traverse in turning round 1152 times?
8. The diameter of a wheel being 3 feet 10 inches, how many yards will it traverse in turning round 60 times?
9. The circumference of a wheel being 1·76 yards, how many times will it turn round in a mile?
10. A wheel is 56 inches high. How many revolutions does it make in 70 miles?
11. The spoke of a bicycle wheel measures 14 inches from centre to rim. How many times will it turn round in travelling half-a-mile?

12. The minute-hand of a school clock being 5 inches long, find the length of the circle its point describes every hour.

13. How many trees at distances of $7\frac{1}{2}$ yards can be planted round a circular field whose radius is 105 yards?

14. The diameter of a circular plantation is 50 yards. What will be the cost of enclosing it with a wall at 2s. 1d. per yard?

15. Find the expense of planting trees around a circular piece of ground whose diameter is 126 feet, the trees being placed one yard apart and the cost of each 1s. 6d.

16. A horse going round a circus ring 20 feet in diameter was observed to make 13 complete circuits in a minute. At what rate (in miles per hour) was he moving?

17. A boy riding on a 'merry-go-round' describes a circle of 14 yards diameter and travels at the rate of 15 miles an hour. How many times does the boy go round in a minute?

100. To find the diameter of a circle, the circumference being given, divide the circumference by $\frac{22}{7}$.

Example.—The circumference of a circle is 82 feet 6 inches; find the diameter.

$$\text{Since } 82 \text{ ft. } 6 \text{ in.} = 82\tfrac{1}{2} \text{ ft.} = \frac{165}{2} \text{ ft.},$$

$$\therefore \text{ Diameter} = \frac{165}{2} \div \frac{22}{7} = \frac{165}{2} \times \frac{7}{22} = \frac{15 \times 7}{4} \text{ feet} = 26 \text{ feet } 3 \text{ inches.}$$

EXERCISE 63.

Find the diameters of the circles whose circumferences are:—

1. 242 feet.
2. 440 yards.
3. 38·5 feet.
4. $9\frac{1}{3}$ inches.
5. 12 feet 10 inches.
6. 3 chains 30 links.

Find the radii of the circles whose circumferences are:—

7. 14 feet.
8. 330 yards.
9. 1 furlong.
10. 19·2 feet.
11. $12\frac{4}{7}$ feet.
12. 27 feet 8 inches.

EXERCISE 64.

1. Find the diameter of a hat to fit a head $23\frac{1}{2}$ inches round.

2. The circumference of the moon being 6850 miles, what is its diameter?

3. What is the radius of a circle whose perimeter is 100 chains?

4. If a carriage-wheel make 600 revolutions in travelling a mile, what is its diameter?

5. If the length of a fence that surrounds a circular field be 12 chains 10 links, find the length of a straight path crossing the widest part of the field.

6. The cost of fencing a circular field at 2s. 1d. per yard being £29 15s. 10d., find the length of a straight path running from side to side through the centre.

101. To find the area of a circle, the diameter and circumference being given.

If we inscribe in a circle a regular polygon with a large number of sides it will be noticed that (1) the area of the polygon does not differ much from the area of the circle; (2) the perimeter of the polygon does not differ much from the circumference of the circle; and (3) the perpendicular drawn from the centre to a side of the polygon does not differ much from the radius of the circle.

If the number of sides be indefinitely increased, it follows that the perimeter of the polygon will be ultimately the circumference of the circle and the perpendicular OP from the centre of the polygon to the side AB will coincide with the radius of the circle (§ 91); **hence the area of a circle is equal to one-half the circumference multiplied by the radius.**

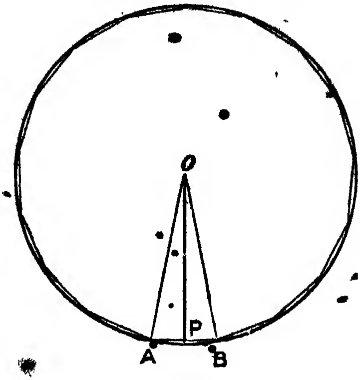


FIG. 83.

Example.—Find the area of a circle whose circumference is 88 feet and radius 14 feet.

$$\text{Since area of circle} = \frac{\text{circumference} \times \text{radius}}{2};$$

$$\therefore \text{ Area} = \frac{88 \times 14}{2} \text{ square feet} = 616 \text{ square feet.}$$

EXERCISE 65.

1. Find the area of a circle whose circumference is 132 feet and radius 21 feet.
2. The diameter of a circle is 12 yards 5 feet and its circumference 39 yards 4 feet. What is its area in square feet?
3. A circular grass-plot is 40 yards 1 foot in circumference and 12 yards 2½ feet across its widest part. Find its area.

4. Find in acres, roods and perches the area of a circular field whose diameter is $115\frac{1}{2}$ poles and circumference 363 poles.

5. Find the area of a circle whose diameter is 7 feet and circumference 22 feet.

6. The circumference of a circular enclosure is 242 yards and its diameter 77 yards. What would be the cost of turfing it at 3s. 4d. per square yard?

102. To find the area of a circle, the diameter being given. Since the circumference is $\frac{22}{7}$ of the diameter (§ 98), half the circumference will be $\frac{11}{7}$ of the diameter. Now, the area of a circle is equal to one half the circumference multiplied by one-half the diameter (§ 101). Hence writing '$\frac{11}{7}$ of the diameter' for '$\frac{1}{2}$ of the circumference,' we have

$$\text{Area of a circle} = \tfrac{11}{7} \text{ of diameter} \times \tfrac{1}{2} \text{ of diameter.}$$
$$= \tfrac{11}{14} \text{ of the square of the diameter.}$$

Hence **the area of a circle is found by mutliplying the square of the diameter by $\frac{11}{14}$.**

Example.—Find the area of a circle whose diameter is 28 feet.

Here area of circle $= \frac{11}{14} \times 28 \times 28$ square feet $= 616$ square feet

EXERCISE 66.

Find the areas of the circles whose diameters are :—

1. 42 feet.
2. 165 feet.
3. 70 feet.
4. 14 yards.
5. 12 yds. 2 ft. 6 in.
6. 3 yds. 2 ft. 8 in.

Find the areas of the circles whose radii are :—

7. 15 feet.
8. 1 foot 9 inches.
9. 2 feet 11 inches.
10. $31\frac{1}{2}$ feet.
11. $5\frac{3}{4}$ feet.
12. $10\frac{1}{2}$ feet.

EXERCISE 67.

1. What is the acreage of a circular plot whose diameter is 200 yards?

2. Find in acres, roods and perches the area of a circular garden 126 yards in diameter.

3. The diameter of a circular park is $115\frac{1}{2}$ poles. Find the area in acres, roods and perches.

4. The diameter of a circular table being 4 feet 3 inches, what is its area?

5. Find in square chains the area of a circular plot whose diameter is 3 chains.

6. A cow is fastened to a post in a paddock by a rope 30 feet long. How much grass is within its reach?

7. Find the expense of turfing a lawn 350 feet in diameter at 3d. per square yard.

8. The diameter of a well being 5 feet, what will a cover of wood cost at $5\frac{1}{2}$d. per square foot?

9. A flat circular roof is 19 feet 6 inches in diameter. What will be the cost of painting it at $4\frac{1}{4}$d. per square foot?

10. If a pressure of 15 lbs. on every square inch be applied to a circular plate 12 feet in radius, what is the total pressure in tons?

11. A garden containing 3 acres has a circular pond in the centre, 100 yards in diameter. Find the area left for cultivation.

12. In the middle of a circular court whose diameter is 112 feet is a square grass-plot whose side is 8 feet. Find the cost of paving the remainder of the court at 1s. 9d. the square yard.

103. To find the area of a circle, the circumference being given.

Since the diameter is $\frac{7}{22}$ of the circumference (§ 98), half the diameter will be $\frac{7}{44}$ of the circumference. Now, the area of a circle is equal to one-half the diameter multiplied by one-half the circumference (§ 101). Hence, writing '$\frac{7}{44}$ of the circumference' for '$\frac{1}{2}$ of the diameter,' we have

$$\text{Area of circle} = \tfrac{7}{44} \text{ of circumference} \times \tfrac{1}{2} \text{ of circumference}$$
$$= \tfrac{7}{88} \text{ of the square of the circumference.}$$

Hence **the area of a circle is found by multiplying the square of the circumference by $\frac{7}{88}$.**

Example.—Find the area of a circle whose circumference is 88 feet.

Here area of circle $= \frac{7}{88} \times 88 \times 88$ square feet $= 616$ square feet.

EXERCISE 68.

Find the areas of the circles whose circumferences are :—

1. 11 ft.
2. 176 ft.
3. 20 ft. 8 in.
4. 355 yds.
5. 58 yds. 2 ft.
6. 40 yds. 1 ft.
7. 13 yds. 1 ft. 4 in.
8. 20 yds. 2 ft. 4 in.
9. 7 chains 4 links.
10. 44 chains.
11. $60\frac{1}{2}$ chains.
12. 960 links.

N.B.—*Give the last three answers in acres, etc.*

EXERCISE 69.

1. A circular garden requires $235\frac{5}{7}$ yards of wire-fencing to enclose it. How many square yards does the garden occupy?

2. A round table is 18·85 feet in circumference. Find the area of its surface in square feet.

3. How much will it cost to turf a circular plot of ground 130 feet in perimeter at 4d. per square yard?

4. The cost of fencing a circular pond at 1s. 6d. per yard is £7 14s. Find the area of the pond.

5. Find the rent of a circular plot of ground whose circumference is 2560 links at £2 per acre.

6. If a carriage wheel turn twice in $16\frac{1}{2}$ feet, and if in passing round a circular cricket ground it make 400 revolutions, what is the area of the ground in acres, etc.?

104. To find the diameter of a circle, the area being given.

Since $\frac{11}{14}\times$ square of the diameter $=$ area (§ 102);
therefore square of the diameter $=$ area $\div\frac{11}{14}$;
therefore diameter $=$ square root of (area $\div\frac{11}{14}$).

Hence to find the diameter, when its area is given,

Divide the area of a circle by $\frac{11}{14}$, and the square root of the quotient is the diameter.

Example.—The area of a circle is 5 sq. yds. 7 sq. ft. 58 sq. in. Find its diameter.

Here 5 sq. yds. 7 sq. ft. 58 sq. in. $=$ 7546 sq. in.

To divide by $\frac{11}{14}$ we multiply by $\frac{14}{11}$.

Now, 7546 sq. in. $\times\frac{14}{11}=9604$ sq. in.;

$\therefore$ Diameter $=\sqrt{9604}$ inches $=98$ inches $=8$ ft. 2 in.

EXERCISE 70.

Find the diameters of the circles whose areas are :—

1. 616 sq. in.
2. 9856 sq. ft.
3. 1 sq. ft. 10 sq. in.
4. 17 sq. yds. 1 sq. ft.
5. 273 sq. yds. 7 sq. ft.
6. 386 sq. ft. 10 sq. in.
7. 2464 sq. ft.
8. 3850 sq. yds.
9. 273 sq. ft. 112 sq. in.
10. 3 sq. yds. $6\frac{4}{7}$ sq. ft.
11. 42 sq. yds. $2\frac{2}{7}$ sq. ft.
12. 8 sq. ch. 1466 links.

EXERCISE 71.

1. Find (1) in chains and links, (2) in yards and feet, the diameter of a circular field which contains an acre.

2. A field in the shape of a semicircle contains 48 acres 20 perches. Find its radius in links.

3. At £90 15s. per acre a circular field cost £36 1s. $10\frac{1}{2}$d. Find the length of the diameter in yards.

4. Find the diameter of a circular plot which shall contain as much ground as a square plot whose side is 10 yards.

5. A circular reservoir having a surface 4 acres in area is to be made. What is its radius in yards?

6. A cow is fastened to a stake so that she can just graze over 1 acre 2706 square yards of grass. Find the length of the cord that fastens her to the stake.

105. To find the circumference of a circle, the area being given.

Since $\frac{7}{88}$ × square of the circumference = area (§ 103);
therefore square of the circumference = area ÷ $\frac{7}{88}$;
therefore circumference = square root of (area ÷ $\frac{7}{88}$).
Hence to find the circumference of a circle, when its area is given,

Divide the area by $\frac{7}{88}$, and the square root of the quotient is the circumference.

Example.—The area of a circle is 2464 square feet. Find its circumference.

To divide by $\frac{7}{88}$ we multiply by $\frac{88}{7}$.

Now 2464 sq. ft. × $\frac{88}{7}$ = 30976 sq. ft.

∴ Circumference = $\sqrt{30976}$ feet = 176 feet.

EXERCISE 72.

Find the circumferences of the circles whose areas are:—

1. 1386 sq. ft.	3. 68 sq. yds. 4 sq. ft.	5. 21 sq. ft. $94\frac{1}{2}$ sq. in.
2. 346 sq. ft. 72 sq. in.	4. 273 sq. yds. 7 sq. ft.	6. 9 sq. yds. 5 sq. ft. 90 sq. in.

EXERCISE 73.

1. Find in chains the circumference of a circular field whose area is 61 acres 2 roods 16 perches.

2. How many yards does a man walk in going round a circular field whose area is 15 acres 1 rood 24 perches?

3. The area of a circular field is 8 acres 2 roods 26 perches. How many minutes will a man take to walk round it at 4 miles per hour?

4. To pave a circular courtyard at 2s. 3d. a square yard cost £17 6s. 6d. What is the distance round the court?

5. A circular fish-pond has an area of 2673 square yards 5 square feet 72 square inches. Find the cost of enclosing it with a fence at 1s. 6d. per yard.

6. A circular bicycle-track has an area of 8 acres 704 square yards. How many times will a bicycle wheel 3 feet 6 inches in diameter turn in going round it?

106. To find the area of a circular ring.

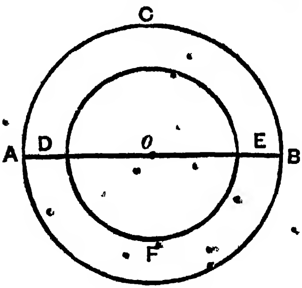


FIG. 34.

When two circles are described from the same centre, but with different radii, they are called **concentric circles** and the space between their circumferences is called a **circular ring**.

The area of a circular ring is evidently equal to the difference between the area of the larger circle *ABC* and of the smaller circle *DEF*.

Example.—The inner and outer diameters of a circular ring are 112 feet and 140 feet respectively. Find the area of the ring.

$$\text{Here area of outer circle} = \frac{11}{14} \times 140 \times 140 \text{ sq. ft.} = 15400 \text{ sq. ft.}$$

$$\text{Area of inner circle} = \frac{11}{14} \times 112 \times 112 \text{ sq. ft.} = 9856 \text{ sq. ft.};$$

$\therefore$ difference between the areas of the circles
$= (15400 - 9856)$ sq. ft. $= 5544$ sq. ft.

EXERCISE 74.

1. The diameter of the inner circle of a ring is 49 feet and of the outer circle 77 feet. Find the area of the ring.

2. Find the area of a circular ring, given that the radius of its outer circumference is 28 feet and that of its inner 21 feet.

3. The outer and inner diameter of a circular ring measure respectively 63 and 42 feet. Find the area of the ring.

4. Find in acres the area of a circular ring whose inner diameter is 14 chains and outer diameter 17 chains 50 links.

5. What is the area of a circular ring 4 feet wide when the diameter of the outer circle is 64 feet?

6. A road passes round a circular piece of ground, the outer and inner circumferences measuring 500 feet and 420 feet respectively. What is the area of the road?

7. A circus ring is 30 feet in diameter and is carpeted for 20 feet from the centre, leaving an outer ring of tan. What is the area of the tan ring?

8. Find the area of a gravel path 1 yard wide which surrounds a circular flower-bed the diameter of which is 4 yards.

9. A road 10 yards wide runs round the circumference of a circular grass plot, and the diameter of the plot is 80 yards. What is the area of the road in acres, etc.?

10. A circular grass-plot whose diameter is 40 yards contains a gravel walk 1 yard wide running round it 1 yard from the edge. What will it cost to turf the gravel walk at 4d. per square yard?

11. In a circular riding-school of 100 feet diameter a circular ride, immediately within the outer edge, is to be made of a uniform width of 10 feet. Find the cost of doing this at 4d. per square foot.

12. The inner diameter of a circular building is $68\frac{6}{7}$ feet, and the thickness of the wall is $1\frac{5}{7}$ feet. Find the number of square feet occupied by the wall.

107. Any part of the circumference of a circle is called an arc, as ABC.

The angle AOC is **the angle subtended by the arc** ABC at the centre O. The length of ABC is called the **length of the arc.**

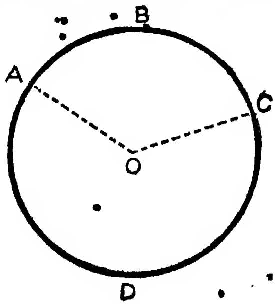


FIG. 35.

108. The circumference of every circle is considered as divided into 360 equal parts, called degrees, marked thus (°). Every degree is divided into 60 equal parts, termed minutes, marked thus (′), and every minute is divided into 60 equal parts, termed seconds, marked thus(″). Hence 36° 45′ 75″ reads 36 degrees 45 minutes 75 seconds.

109. Now it is clear that the length of the arc ABC bears the same ratio to the circumference of the circle $ABCD$ that the number of degrees in the angle AOC does to 360°. Thus, if

the angle subtended by the arc be 90°, or one-fourth of 360°, the length of the arc will be one-fourth of the circumference; if the angle be 30°, or one-twelfth of 360°, the length of the arc will be one-twelfth of the circumference; and so on.

110. To find the length of the arc of a circle when the circumference of the circle and the number of degrees in the angle subtended by the arc at the centre are given.

Example.—The circumference of a circle is 48 inches, and the angle subtended by an arc at the centre is 45°. Find the length of the arc.

$$\text{Here length of } 360° = 48 \text{ inches.}$$

$$\therefore \quad ,, \quad 1° = \frac{48}{360} \text{ inches.}$$

$$\therefore \quad ,, \quad 45° = \frac{48 \times 45}{360} \text{ inches} = 6 \text{ inches.}$$

NOTE.—When the radius or diameter of the circle is given, first find the circumference (§ 99).

EXERCISE 75.

1. The circumference of a circle is 45 inches and the angle subtended by an arc at the centre is 96°. Find the length of the arc.
2. What is the length of an arc of 50° in a circle whose circumference is 30 inches?
3. Find the length of an arc of 45°, the radius of the circle being 28 yards.
4. What is the length of an arc of 30° in a circle whose diameter is 3 feet 6 inches?
5. The radius of a circle is 2 feet $3\frac{1}{2}$ inches and the angle subtended by the arc at the centre is 75°. Find the length of the arc in feet.
6. What is the length of an arc of 45° in a circle whose diameter is 16 feet 4 inches?
7. The circumference of a circle is 4 feet and the angle subtended by an arc at the centre is 37° 30′. Find the length of the arc in inches.
8. The diameter of a circle being 40 feet and the angle subtended by the arc at the centre 25° 30′, find the length of the arc.
9. What is the length of an arc of 78°45′ in a circle whose radius is 4 feet 8 inches?

111. To find the number of degrees in the angle subtended by the arc at the centre of a circle when the circumference of the circle and the length of the arc are given.

Example.—The circumference of a circle is 176 inches. Find the

number of degrees in the angle subtended at the centre by an arc of 22 inches.

Since number of degrees in the whole circumference $= 360$;

$\therefore$,, ,, in arc of 1 inch $= \frac{360}{176}$.

$\therefore$,, ,, in arc of 22 inches $= \frac{360 \times 22}{176}$.

$= 45$.

NOTE.—When the radius or diameter of the circle is given, first find the circumference (§ 99).

EXERCISE 76.

1. The circumference of a circle is 14 ft. 8 in., and the length of the arc 1 ft. 10 in. Find the angle subtended at the centre by the arc.
2. The circumference of a circle is 30 feet. Find the number of degrees in an arc whose length is 15 inches.
3. The diameter of a circle is 70 feet. How many degrees are subtended at the centre by an arc 22 feet long?
4. The arc of a circle is 5 ft. 6 in.; the diameter of the circle is 8 ft. 9 in. Find the angle subtended at the centre by the arc.
5. Find the angle subtended by an arc of 44 inches in a circle whose radius is $52\frac{1}{2}$ inches.
6. The radius of a circle is 4 ft. 8 in., and the length of the arc is 6 ft. 5 in. Find the angle subtended at the centre by the arc.
7. The length of an arc is 28·8 feet and the radius of the circle 84 feet. Find the angle subtended by the arc at the centre.
8. Find the angle subtended by an arc 10 inches long in a circle whose radius is 10 inches.

112. It has been stated (§ 107) that any part of the circumference of a circle is termed an arc.

Now if two straight lines be drawn from the extremities of the arc to the centre of the circle, the surface thus enclosed is called a **sector** of a circle.

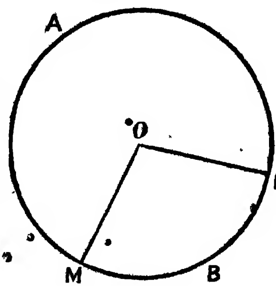


FIG. 36.

Thus *MONB* is a sector of the circle *AMBN* bounded by the two radii *OM* and *ON* and the arc *MBN* between them.

MON is the **angle of the sector,** or the angle subtended by the arc *MBN* at the centre.

The remaining portion *MONA* of the circle is also a sector.

113. Now it is clear that the area of the sector *MONB* bears the same ratio to the area of the circle that the number of degrees in *MON* does to 360°.

114. To find the area of a sector of a circle when the area of the circle and the angle of the sector are given.

Example.—Find the area of the sector of a circle whose radius is 14 inches, and whose arc subtends at the centre an angle of 45°.

The circle can evidently be divided into 360 sectors, each having an arc of 1°.

Here area of whole circle or 360° = 616 sq. inches (§ 102).

$\therefore$ Area of sector of 1° $= \frac{616}{360}$ sq. inches.

$\therefore$,, ,, of 45° $= \frac{616 \times 45}{360}$ sq. inches.

$= 77$ sq. inches.

EXERCISE 77.

1. The area of a circle is 7546 square feet; the angle of the sector is 60°. Find the area of the sector in square yards, etc.

2. The radius of a circle is 35 feet; the angle subtended by the arc at the centre is 36°. Find the area of the sector.

3. The diameter of a circle is 56 feet; the angle which the arc subtends at the centre is 45°. Find the area of the sector.

4. The diameter of a circle is 30 feet; the angle of the sector is 30°. Find the area of the sector in square feet.

5. Find the area of a sector of a circle whose radius is $8\frac{3}{5}$ feet and whose arc contains 40°.

6. The radius of a circle is $24\frac{1}{2}$ feet; the angle subtended by the arc 135°. Find the area of the sector in square yards, etc.

7. If the diameter of a circle be 50 feet and the angle of the sector 157° 30′, find the area of the sector in square yards, etc.

8. The area of a sector is 385 square feet; the angle of the sector is 36°. What is the radius of the circle?

9. The area of the sector of a circle is 86 square feet 90 square inches; the angle of the sector is 40°. Find the radius of the circle.

10. The area of a sector is 235 square feet 117 square inches; the angle of the sector is 45°. Find the radius of the circle.

11. The area of a sector is 462 square feet and the radius of the circle is 42 feet. What is the angle of the sector?

12. The area of a sector is 28 square yards 7 square feet 112 square inches and the diameter of the circle is 21 yards. Find the number of degrees in the angle of the sector.

115. To find the area of a sector of a circle when the radius of the circle and the length of the arc are given.

Since the area of a circle is equal to one half the circum-

ference multiplied by the radius (§ 101), therefore the area of a sector, which is a portion of a circle, may be found by multiplying half its circumference, *i.e.* the length of the arc which subtends the angle at the centre, by the radius of the circle.

Example.—The radius of a circle is 2 ft. 11 in.; the length of an arc of a sector is 16 ft. 6 in. Find the area of the sector.

$$\text{Area of sector} = \left(\frac{1}{2} \times \frac{33}{2} \times \frac{35}{12}\right) \text{ square feet}$$
$$= \frac{1155}{48} \text{ sq. feet} = 24 \text{ sq. feet } 9 \text{ sq. inches.}$$

EXERCISE 78.

1. The radius of a circle is 15 feet; the length of an arc of a sector 12 feet. Find the area of the sector.

2. The length of an arc of a sector is 8 feet and the radius is $3\frac{1}{2}$ feet. Find its area.

3. Find the area of a sector when the arc of the sector is 42 feet and the radius of the circle is equal to the length of the arc.

4. What is the area in square yards, etc., of a sector of a circle whose radius measures $17\frac{1}{2}$ feet, if the arc measure 37 feet?

5. Find the radius of a circle, if the area of a sector whose arc is 41 feet long be 41 square yards.

6. Find the radius of a circle, if the area of a sector whose arc measures 1 foot 6 inches be 2 square feet 90 square inches.

116. It has been already stated that any part of the circumference of a circle is termed an arc.

The straight line which joins the ends of an arc is called a **chord.** Thus *AB* is a chord of the arc *AMB*.

The part of a circle which is cut off by the chord of the arc is called a **segment.** Thus in the circle *AMBN* the figure *AMB* is a segment less than a semicircle, being bounded by the chord *AB* and the arc *AMB*. The remaining portion of the circle *ANB* is also a segment greater than a semicircle.

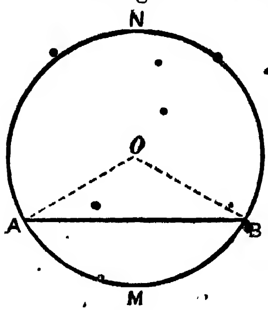


Fig 37.

Note.—Every chord, except the diameter, divides the circle into two unequal segments.

117. From the above figure it is evident that

(1) The sector $AOBM$ and the segment AMB have the same arc AMB.

(2) The area of the segment AMB equals the area of the sector $AOBM$, *minus* the area of the triangle AOB.

(3) The sector $AOBN$ and the segment ANB have the same arc ANB.

(4) The area of the segment ANB equals the area of the sector $AOBN$, *plus* the area of the triangle AOB.

118. To find the area of a segment of a circle.

Find the area of its corresponding sector; then, if the segment be *less* than a semicircle *subtract* the area of the triangle from the area of the sector; if the segment be *greater* than a semicircle *add* the area of the triangle to the area of the sector.

EXERCISE 79.

Revision Examination.—The Circle.

1. What is the cost of putting a curb-stone $1\frac{1}{2}$ feet wide round a well 9 feet in diameter, at 1s. 3d. per square foot?

2. Find the cost of paving a semicircular courtyard 98 yards in diameter at 3d. per square foot.

3. A road passes round a circular plot of ground, the outer circumference of which is 600 feet and the inner 480 feet. Find the breadth of the road.

4. What is the acreage of a field in the form of a sector of a circle, if the angle be 120° and the diameter of the circle 252 yards?

5. A man stands in the middle of a circus ring, whose area is 154 yards, holding a cord attached to a horse running round the ring. What is the greatest length of the rope?

6. The driving-wheel of a railway engine has a diameter of 6 feet 5 inches. How many times will it turn in a minute if the engine be running at a speed of 55 miles per hour?

7. A bicycle wheel is $3\frac{1}{2}$ feet in diameter and turns exactly 168 times in going round a circular bicycle track. How many square yards are there in the plot enclosed by the track?

8. The pit of a theatre is semicircular and its greatest length is 40 feet. How many spectators will it hold if a space of 4 feet 6 inches be allotted to each?

9. How many square feet are there in the area of the surface of a kite whose length is 6 feet, that being three times the radius of its semicircular head?

10. If the string of a kite, being 150 yards long, is curved into a semicircle, what is the actual distance from the kite to the hand of the holder of the string?

Part II.

MENSURATION OF SOLIDS.

CHAPTER VIII.

RECTANGULAR SOLIDS.

119. A **solid** body has three dimensions, namely, length, breadth and thickness (height or depth); *e.g.*, a log of wood, a block of stone, or a box.

120. Amongst all the bodies that can be imagined, a **cube** is one of the most simple, having its three dimensions all equal, and each of its six faces a square.

121. The space taken up by a solid is its **volume, solidity,** or **solid content.**

This volume is measured by the number of times it contains some standard volume.

122. If the cube taken as the unit of measurement be one inch in length, one inch in breadth, and one inch in thickness, it is called a *cubic inch*; if its dimensions be each one foot, it is called a *cubic foot*; if its three dimensions be each a yard, it is called a *cubic yard.*

123. The solidity of a body is therefore often called its **cubical content.**

124. **To find the volume of a cube.**

The figure annexed is a solid or cubic inch, having the length AB, the breadth BC and the height CD each one inch. AB, BC, CD, etc., are called the *edges* or *sides* of the cube.

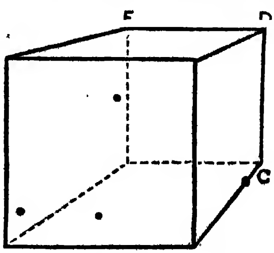


Fig. 38.

If 144 of these cubic inches were arranged in 12 rows, each containing 12 cubes, a solid would be formed, one inch high, having a base whose area would be a square foot.

If twelve such solids were piled one above another, a cube would be formed whose length, breadth, and height would be each a

foot. Since (144 × 12) cubic inches have been used, it is evident that

1728 cubic inches make 1 cubic foot.

In the same manner it may be shown that

27 cubic feet make 1 cubic yard.

Hence the number of cubic inches (or cubic feet, etc.) in the volume of a cube is found by taking the product of the number of inches (or feet, etc.) in the height by the number of square inches (or square feet, etc.) in the base; or briefly by cubing the number of inches (or feet, etc.) in its edge.

Example 1. Find the volume of a cube whose edge is 8 inches long.

Volume = (8 × 8 × 8) cubic inches = 512 cubic inches.

Example 2. Find the solidity of a cube whose side is 3 feet 6 inches.

Volume $= (3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2})$ cubic feet $= \left(\frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right)$ cubic feet $=$

$\frac{343}{8}$ cubic feet $= 42\frac{7}{8}$ cubic feet $=$ 42 cubic feet 126 cubic inches.

NOTE.—The term *side* of a cube denotes not one of its side faces but one of its edges.

EXERCISE 80.

Find the volume of the cubes whose edges are respectively :—

1. 13 yards.	5. $2\frac{1}{3}$ yards.	9. 2 feet 9 inches.
2. 21 yards.	6. $10\frac{5}{12}$ feet.	10. 3 feet 10 inches.
3. 1·6 yards.	7. 8 feet.	11. 5 feet 6 inches.
4. 4·5 yards.	8. 13 feet.	12. 6 feet 8 inches.

EXERCISE 81.

1. How many cubic feet of earth were removed in digging a cubical cellar whose length is 16 feet?

2. How many cubic feet of air are there in a room whose length, breadth and height measure each 21 feet?

3. A cubical box measures 6 feet in length; how many cubic inches does it contain?

4. A cubical-shaped tank measures 2 feet 3 inches along the inner side; how many cubic feet of water will it hold?

5. A cubical mass of wood is $2\frac{1}{2}$ inches long; what is its value at 10d. per cubic inch?

6. What is the solid content of a cube, in cubic yards, etc., whose length, breadth, and depth are each 12 feet 6 inches?

125. Since the volume of a cube is found by taking the product of the length, breadth and height; (in other words, by cubing the length of its edge) it follows that **the volume of a cube being known, the number of linear units in the edge is found by taking the cube root of the number of cubic units.**

Example. A cube contains 330 cubic feet 1547 cubic inches; find its edge.

Volume of cube $= 330 \times 1728 + 1547$ cubic inches.

$\therefore$ Length of edge $= \sqrt[3]{571787}$ inches
$= 83$ inches
$= 6$ feet 11 inches.

EXERCISE 82.

Find the length of the edge of the following cubes whose volumes are respectively :—

1. 5832 cub. inches.
2. 13824 cub. inches.
3. 32768 cub. inches.
4. 110592 cub. inches.
5. 17·576 cub. feet.
6. 493·039 cub. yards.
7. $49\frac{8}{27}$ cub. feet.
8. $181\frac{78}{81}$ cub. yards.
9. 15 cub. ft. 1080 cub. in.
10. 42 cub. ft. 1512 cub. in.
11. 11 cub. ft. 675 cub. in.
12. 37 cub. ft. 64 cub. in.

EXERCISE 83.

1. A solid block of marble contains 15625 cubic feet; what is the length of each of its sides?
2. A box contains 19683 cubic inches; find the length of its side.
3. A cubical cistern contains 190 cubic feet 189 cubic inches of water; find the length of its side.
4. A seaman has a chest made in a cubical form and which contains 76 cubic feet 1323 inches of space; what is the length of its inner edge?
5. A packing case in the shape of a cube containing 512 cubic feet is exactly filled by 32 cubical boxes; find the length of the side of each box.
6. The volume of a cubical block of granite being $3605\frac{1}{27}$ cubic feet, find the length of its edge in feet and inches.

126. The surface of a solid is its outside area, and since the surface of a cube consists of six faces, each of which is a square, **the area of the whole surface of a cube is equal to the sum of the areas of the squares which bound it.**

Example. Find the whole surface of a cube whose edge is 9 inches.

Here area of each face $= (9 \times 9)$ square inches $= 81$ square inches.

$\therefore$ Area of whole surface $= 81$ square inches $\times 6 = 486$ square inches.
$= 3$ square feet 54 square inches.

EXERCISE 84.

Find the area of the whole surface of the cubes whose edges are respectively :—

1. 20 feet.
2. 8 inches.
3. 3 feet 4 inches.
4. 1 foot 3 inches.
5. 4 feet 2 inches.
6. 10 feet 7 inches.

EXERCISE 85.

1. Find the expense of painting the surface of a cube whose side is 4 feet 6 inches at 1s. 8d. per square yard.

2. Find the expense of covering a cube whose edge is 2 feet 4 inches with copper at 10½d. per square foot.

3. Find the expense of covering a box whose edge is 2¼ feet with lead at £2 2s. per cwt., assuming that 6 lb. of lead will cover a square foot.

4. If the solid content of a cubical block of stone be 37 cubic feet 64 cubic inches, what is the area of its sides?

5. The content of a cubical shaped box being 3 cubic feet 648 cubic inches, find the cost of painting its sides at 6d per square yard.

6. A cubical block of wood contains 1 cubic yard 2 cubic feet 541 cubic inches; find the cost of painting its entire surface at 4s. 6d. per square yard.

7. The content of a cubical-shaped box being 650·962 cubic feet, find the cost of covering it with cloth 2 feet 2 inches wide at 9d. per yard.

8. The area of the whole surface of a cube is 104 square feet 2 square inches; find the length of an edge.

9. The area of one side of a cubical cistern contains 12 square feet 36 square inches; find the capacity of the cistern.

10. The area of the whole surface of a cube is 11094 square inches; find the volume of the cube.

127. It has been already stated that a solid bounded by six equal square faces is called a cube. A solid bounded by six rectangular faces, each of them equal and parallel to its opposite face, is called a **rectangular parallelopiped.**

A common match-box and an ordinary bar of soap are familiar examples.

The annexed figure represents a parallelopiped in which it will be readily seen that the following faces are equal and parallel:—

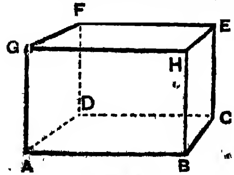


Fig. 39.

(1) $ABHG$ and $DCEF$ (front and back);
(2) $GHEF$ and $ABCD$ (top and bottom);
(3) $ADGF$ and $BCHE$ (two ends).

$ABCD$ is called the base; AB or GH the length; GF or HE the breadth; and AG or DF the height or depth.

128. To find the volume of a rectangular parallelopiped. Suppose the length of the rectangular parallelopiped to be 5 inches, the breadth 4 inches and the depth 3 inches. Divide the edges respectively into 5, 4 and 3 equal parts, each being one inch. Draw parallel lines through the points of division parallel to the outer faces of the solid, as in the diagram. Then the solid is divided into a number of smaller blocks each of which is a cubic inch.

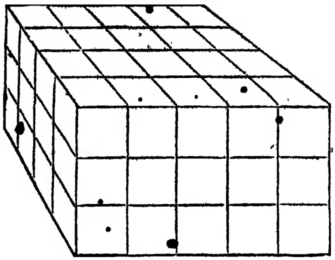


Fig. 40.

The number of cubes (20) in each horizontal row will be the product of the number of inches in the length (5) by the number of inches in the breadth (4), and the number of these rows (3) will be equal to the number of inches in the height. Thus, in this case the number of cubic inches in the whole solid will be $(5 \times 4 \times 3)$ cubic inches, or the volume $=60$ cubic inches. Hence, **the number of cubic inches (or cubic feet, etc.) in the volume of a rectangular parallelopiped is equal to the continued product of the number of inches (or feet, etc.), in its length, breadth and thickness.**

Example.—Find the volume of a rectangular parallelopiped 4 feet 8 inches long, 2 feet 9 inches broad and 1 foot 6 inches deep.

$$\text{Volume} = (4\tfrac{2}{3} \times 2\tfrac{3}{4} \times 1\tfrac{1}{2}) \text{ cubic feet.}$$
$$= \left(\frac{14}{3} \times \frac{11}{4} \times \frac{3}{2}\right) \text{ cubic feet.}$$
$$= 19\tfrac{1}{4} \text{ cubic feet} = 19 \text{ cubic feet } 432 \text{ cubic inches.}$$

EXERCISE 86.

Find the volumes of the rectangular parallelopipeds which have the following dimensions:—

1. Length, 13 ft. 6 in.; breadth, 8 ft. 4 in.; depth, 6 ft.
2. Length, 6 ft. 8 in.; breadth, 5 ft. 7 in.; depth, 4 ft. 6 in.
3. Length, 15 ft. 6 in.; breadth, 3 ft.; depth, 2 ft. 10 in.
4. Length, 5 ft. 6 in.; breadth, 4 ft. 5 in.; depth, 3 ft. 4 in.
5. Length, 2 ft. 9 in.; breadth, 1 ft. 8 in.; depth, 1 ft. 4 in.
6. Length, 32 ft. 7 in.; breadth, 2 ft. 11 in.; depth, 2 ft. 5 in.

EXERCISE 87.

1. How many cubic inches are there in a piece of wood 7 inches long, $3\frac{1}{2}$ inches broad and $1\frac{3}{4}$ inches thick?

2. How many cubic feet of air are there in a room 120 feet long, $62\frac{1}{2}$ feet wide and $25\frac{2}{3}$ feet high?

3. How many cubic yards of earth were removed in excavating the foundation of a house 72 feet long, 45 feet broad and 4 feet deep?

4. How many cubic feet of brickwork are there in a wall 427 feet 6 inches long, 13 feet 4 inches high and 1 foot 2 inches thick?

5. How many loads (cubic yards) of gravel would be required to cover to a depth of 2 inches a path 90 yards long and 5 feet wide?

6. The inside of a rectangular water-cart measures 4 feet 7 inches in length, 3 feet 8 inches in width and 3 feet in depth; what volume of water will it hold?

7. How many cubic feet of lead $\frac{2}{15}$ inch thick will be required to cover the steps of a staircase, composed of 50 steps, each 2 feet long and 8 inches wide?

8. The area of the base of a cistern is 3 square feet; how many cubic feet of water must be drawn off to make the surface sink 1 foot 8 inches?

9. What is the value of a block of stone 8 feet 4 inches long, 2 feet 6 inches wide and 1 foot 3 inches thick, at 10d. per solid foot?

10. Find the cost of excavating a cellar 6 yards long, 16 feet wide and 7 feet deep, at 9d. per cubic yard.

11. Find the cost of the flooring of a room consisting of 36 planks, each $10\frac{1}{2}$ feet long, 8 inches wide and 3 inches thick, if a cubic foot of timber be worth 1s. $7\frac{1}{2}$d.

12. What is the value of a solid block of metal 6 feet 9 inches long, 6 feet 1 inch wide and 5 feet 4 inches deep, at 3s. 4d. per cubic foot?

129. To find how many times one volume is contained in another volume.

Example.—How many bricks 9 inches long, $4\frac{1}{2}$ inches wide and 3 inches deep, will be required to build a wall 90 feet long, 18 inches thick and 8 feet high?

Here Volume of wall $= (90 \times 12 \times 18 \times 8 \times 12)$ cubic inches,
and Volume of each brick $= (9 \times 4\frac{1}{2} \times 3)$ cubic inches.

$$\therefore \text{ No. of bricks required} = \frac{90 \times 12 \times 18 \times 8 \times 12}{9 \times 4\frac{1}{2} \times 3}$$
$$= 15360.$$

EXERCISE 88.

1. How many four-inch cubes can be cut out of a rectangular piece of timber 15 feet 4 inches long, 2 feet 10 inches broad and 1 foot 8 inches thick?

2. How many children will a schoolroom, whose floor is 60 feet by 40 feet, and whose height is 15 feet, accommodate, allowing 100 cubic feet of air for each child?

3. How many tiles 6 inches square and 1 inch thick can be put together in a pile 38 feet long, 10 feet 6 inches wide and 6 feet 5 inches high?

4. A wall is 15 feet 8 inches long, 11 feet 6 inches high and 11 inches thick, and has a doorway 6 feet 3 inches high by 2 feet 4 inches wide. Find the number of bricks contained in it, if each brick contains $165\frac{7}{12}$ cubic inches.

5. What will a brick wall cost to build—5 feet high, $22\frac{1}{2}$ inches thick and 35 yards long, if each brick measures 9 inches by $4\frac{1}{2}$ inches by 3 inches and the bricks cost 15s. per thousand?

6. A man tries to pack in a case 5 feet 3 inches long, 3 feet wide and 2 feet 9 inches high, 100 dozen of books, each of which is $10\frac{1}{2}$ inches long, $4\frac{1}{2}$ inches broad and $1\frac{3}{8}$ inches thick. How many must he leave out?

130. The **unit of volume** is a cubic inch, and it has been found by actual experiment that such a cube of pure water weighs 252·458 grains.

∴ One cubic foot of pure water weighs 252·458 grains × 1728.

But an avoirdupois pound weighs 7000 grains.

∴ A cubic foot of pure water weighs $\dfrac{252{\cdot}458 \times 1728}{7000}$ lbs. avoir.

$$= \frac{252{\cdot}458 \times 1728 \times 16}{7000} \text{ ozs.} = 997{\cdot}13 \text{ ozs.}$$

This is so near 1000 *ozs. that a cubic foot of water is generally considered to weigh* 1000 *ozs., or* $62\frac{1}{2}$ *lbs.*

Again, by actual experiment, it has been found that a gallon of water weighs 10 lbs. = 7000 grains × 10.

But one cubic inch of pure water weighs 252·458 grains.

∴ Number of cubic inches occupied by one gallon of water $= \dfrac{10 \times 7000}{252{\cdot}458}$

$= 277{\cdot}274.$

This result is so near $277\frac{1}{4}$, *that a gallon of water is generally considered to occupy* $277\frac{1}{4}$ *cubic inches.*

Example.—A cistern is 8 feet long, 4 feet wide and 2 feet deep; how many gallons will it hold, if a cubic foot of water weighs 1000 ounces, and a gallon of water weighs 10 lb.?

The cistern will hold (8 × 4 × 2) cubic feet. Hence the weight of the water which will fill the cistern is 1000 ounces × 8 × 4 × 2.

Now a gallon weighs 160 ounces;

$$\therefore \text{No. of gallons required} = \frac{1000 \times 8 \times 4 \times 2}{160}$$

$$= 400.$$

EXERCISE 89.

1. What is the weight of the water (in tons, etc.) in a cistern whose length, breadth and depth are respectively 12 feet, 6 feet and 3 feet, if a cubic foot of water weigh 1000 ounces?

2. A cistern is 4 feet long, 2 feet 6 inches broad and 3 feet 3 inches deep; what weight of water will it contain (in cwts., etc), if a cubic foot of water weigh 1000 ounces?

3. How many tons of water are there in a reservoir 7 yards long, 10 feet 8 inches wide and 4 feet deep?

4. How many gallons of water will a cistern contain 4 feet long, 3 feet broad, and $2\frac{1}{2}$ feet deep, assuming a cubic foot of water to contain 6·25 gallons?

5. How many gallons of water will a cistern hold whose length is 17 feet 2 inches, breadth 6 feet 5 inches, and depth 4 feet 3 inches, if a gallon contain 277·274 cubic inches?

6. Find the weight (in tons) of a rectangular solid 10 feet by 8 feet by 7 inches, assuming a cubic inch to weigh 2 ounces (avoirdupois).

7. If a pond an acre in area is covered with ice 6 feet thick, and a cubic foot of ice weighs 928 ounces (avoirdupois), find the whole weight of ice in tons, etc.

8. The weight of a cubic foot of water is 1000 ounces, and marble is 2·7 times heavier than water; find the weight of a cubical block of marble whose side measures 5 feet.

131. To find the cubic feet (or cubic inches) of the material used in the construction of a rectangular box or cistern.

Example.—The external dimensions of a closed box are, length 4 feet, breadth 2 feet, and depth 1 foot 4 inches, and the wood of which it is made is an inch thick; how many cubic feet and inches of wood in it?

The internal length $= 4$ feet $- (1$ inch $\times 2) = 3$ ft. 10 in.
" " breadth $= 2$ feet $- (1$ inch $\times 2) = 1$ ft. 10 in.
" " depth $= 1$ ft. 4 in. $- (1$ inch $\times 2) = 1$ ft. 2 in.

Now the volume of the wood is the difference between the volume given by the external and the interior dimensions.

Hence volume required $= (4 \times 2 \times 1\frac{1}{3} - 3\frac{5}{6} \times 1\frac{5}{6} \times 1\frac{1}{6})$ cub. feet.
$= 10\frac{2}{3} - 8\frac{43}{216}$ cub. feet.
$= 2\frac{101}{216}$ cub. feet $= 2$ cub. feet 808 cub. inches.

EXERCISE 90.

1. The external dimensions of a closed box are length 6 feet 2 inches, breadth 3 feet 8 inches, and depth 2 feet; the wood of which it is made being an inch thick, find the number of cubic feet and inches of wood.

2. The external dimensions of a box without a lid are length 4 feet, breadth 3 feet, and depth 2 feet; the wood of which it is made being an inch thick, find the number of cubic feet and inches of wood.

3. A box without a lid is made of wood 1 inch thick; its external dimensions are length, 3 feet 6 inches, breadth 3 feet, depth 2 feet; required the number of cubic feet and inches of wood in it.

4. A cistern without a lid is made of lead 3 inches thick; its external dimensions are 3 feet 3 inches, 2 feet 6 inches and 2 feet; find the number of cubic feet and cubic inches of lead required.

5. The external dimensions of a closed box are length 2 feet, breadth $1\frac{1}{2}$ feet, and height 1 foot. How many cubic inches of air does it contain, if the walls of the box are $1\frac{1}{2}$ inches thick?

6. A cistern is 24 feet long, 18 feet broad, and 12 feet deep; the thickness of the iron is 18 inches; how many cubic feet of water will the cistern hold?

132. Duodecimals may be used in calculating the solidity or volume of bodies.

Example.—Find by duodecimals the volume of a rectangular solid 11 ft. $1\frac{1}{2}$ in. long, 6 ft. $6\frac{2}{3}$ in. broad, and 11 ft. 8 in. high.

	ft.	′	″		
Length =	11	1	6		
breadth =	6	6	8		
	66	9	0		
	5	6	9	0‴	
		7	5	0	0⁗
area of base =	72	11	2		
height =	11	8			
	802	2	10		
	48	7	5	4	
volume =	850	10′	3″	4‴	
=	850 cub. ft. 1480 cub. in.				

EXERCISE 91.

Express each answer in cubic feet and cubic inches.

Find, by duodecimals, the volume of rectangular solids which have the following dimensions:—

1. Length, 3 ft. 2 in.; breadth, 5 ft. 7 in.; height, 6 ft. 8 in.
2. Length, 17 ft. 3 in.; breadth, 14 ft. 7 in.; height, 11 ft. 2 in.
3. Length, 18 ft. 4 in.; breadth, 14 ft. 3 in.; height, 12 ft. 9 in.
4. Length, 12 ft. 6′; breadth, 5 ft. 3′; height, 8 ft. 9′
5. Length, 45 ft. 8′; breadth, 19 ft. 1′ 6″; height, 6 ft. 9′
6. Length, 11 ft. 1′ 6″; breadth, 6 ft. 6′ 8″; height, 11 ft. 8′
7. Length, 7 ft. 6 in.; breadth, 10 ft. $1\frac{1}{2}$ in.; height, 12 ft. $4\frac{1}{2}$ in.
8. Length, 37 ft. $5\frac{1}{2}$ in.; breadth, 22 ft. $7\frac{3}{4}$ in.; height, 10 ft. 6 in.

133. Since the volume of a rectangular parallelopiped is equal to the continued product of the length, breadth, and thickness (or depth), it follows that if the volume and two other dimensions be known, the other dimension can be at once found. This can be expressed briefly thus :—

Since **length × breadth × depth = volume ;**

$$\text{(1) } \textbf{length} = \frac{\textbf{volume}}{\textbf{breadth} \times \textbf{depth}},$$

$$\text{(2) } \textbf{breadth} = \frac{\textbf{volume}}{\textbf{length} \times \textbf{depth}},$$

$$\text{(3) } \textbf{depth} = \frac{\textbf{volume}}{\textbf{length} \times \textbf{breadth}}.$$

Example.—The solid content of rectangular parallelopiped is $167\frac{1}{2}$ cubic feet; its breadth is 5 ft. 7 in. and its depth 4 ft. 6 in. ; find its length.

$$\text{Since length} = \frac{\text{volume}}{\text{breadth} \times \text{depth}},$$

$$\therefore \text{ Length required} = \frac{167\frac{1}{2}}{5\frac{7}{12} \times 4\frac{1}{2}} \text{ feet.}$$

$$= \left(\frac{335}{2} \times \frac{12}{67} \times \frac{2}{9}\right) \text{ feet.}$$

$$= 6\tfrac{2}{3} \text{ feet} = 6 \text{ ft. } 8 \text{ in.}$$

EXERCISE 92.

Find the third dimension of the following rectangular parallelopipeds, when the volume and the other two dimensions are respectively :—

1. Volume, 6 cubic yards ; breadth, 6 ft. ; depth, 3 ft.
2. Volume, 294·25 cub. ft.; length, 9·75 ft., breadth, 6 ft.
3. Volume, $438\frac{3}{4}$ cub. ft. ; length, 13 yds. ; breadth, 9 yds.
4. Volume, 7 cub. ft. 864 cub. in. ; length, 2 ft. 6 in. ; depth, 1 ft. 4 in.
5. Volume, 18 cub. yds. 4 cub. ft. ; breadth, 2 yds. 1 ft. ; depth, 5 ft.
6. Volume, 6 cub. yds. 1 cub. ft. 216 cub. in. ; length, 30 ft. ; depth, 2 ft. 3 in.

EXERCISE 93.

1. What length (in yards) must be cut off a beam $13\frac{1}{2}$ inches broad and 8 inches thick, that it may contain 18 cubic feet ?

2. Ten loads (cubic yards) of gravel are spread uniformly over a path 180 feet long and 4 feet wide ; what is the depth of gravel ?

3. How high must a room be, whose length and breadth are respectively 31 feet 3 inches and 24 feet, in order that it may contain 10000 cubic feet of air ?

4. A rectangular cistern is 13 yards long and 9 feet broad ; $438\frac{3}{4}$ cubic feet of water are drawn off ; find how many inches the surface has sunk.

5. Find the area of a floor of a room, if the room is 8 feet 10 inches high and contains 1855 cubic feet of air.

6. The cubic content of a block of stone is 3 cubic yards 3 cubic feet and the area of its base is 24 square feet; what is its height?

7. A rectangular block of stone has a square base each side of which measures 2 feet $4\frac{1}{2}$ inches; it contains $62137\frac{1}{2}$ cubic inches; find the height.

8. The cubic content of a square tank is 450 cubic yards and the depth is 6 feet; find the length (in yards).

9. If a wall be covered with plaster $1\frac{1}{2}$ inches deep; how many square yards can be covered by a cubic yard of plaster?

10. The beams of wood used in building a house are 3 inches thick and 10 inches wide. If 200 of them are used, which together amount to 1000 cubic feet, what is the length of each beam?

11. Find the thickness and solid content of an armour plate 22 feet by 7 feet, weighing 34 tons, having given that $5\frac{1}{2}$ cubic feet of iron weigh 1 ton.

12. If a cistern 8 feet by 4 feet contain 400 gallons, find the depth of the water, having given that a gallon weighs 10 lbs. and a cubic foot of water 1000 ozs.

134. The surface of a rectangular parallelopiped consists of six rectangles; therefore **the area of the whole surface of a rectangular parallelopiped is equal to the sum of the areas of the six faces.**

Example.—Let *ABCEFG* be a rectangular parallelopiped (§ 127) and let $AB=6$ inches, $GF=4$ inches, and $AG=2$ inches. Find the area of the whole surface.

Surface of top and bottom $=6\times4\times2=48$ sq. inches.
,, front and back $=6\times2\times2=24$ sq. inches.
,, two ends $=4\times2\times2=16$ sq. inches.
∴ Area of whole surface $=(48+24+16)$ sq. inches $=88$ sq. inches.

NOTE.—The opposite sides being equal, there are two sides whose area is the product of the length and width; two sides whose area is the product of the length and height, and two sides whose area is the product of the width and height.

EXERCISE 94.

Find the whole surface of the following rectangular parallelopipeds whose dimensions are respectively:—

1. Length, 15 inches; breadth, 14 inches; depth, 8 inches.
2. Length, 1 ft. 6 in.; breadth, 1 ft. 3 in.; depth, 10 in.
3. Length, 6 ft. 4 in.; breadth, 5 ft. 6 in.; depth, 3 ft. 6 in.
4. Length, 15 ft. 6 in.; breadth, 8 ft. 4 in.; depth, 5 ft. 6 in.
5. Length, 6·25 yds.; breadth, 5·5 yds.; depth, 7·5 feet.
6. Length, $5\frac{3}{4}$ feet; breadth, $3\frac{1}{3}$ feet; depth, $2\frac{1}{2}$ feet.

EXERCISE 95.

1. What will it cost to paint the outside of a box 18 feet long, 12 feet broad, and 8 feet 6 inches deep at $9\frac{1}{2}$d. per square yard ?

2. What will it cost to cover with zinc the sides of a cistern 7 feet 10 inches long, 5 feet 4 inches broad, and 1 foot 9 inches deep at 6s. 9d. per square yard ?

3. Find the cost of painting a closed box 2 feet 3 inches high, 3 feet 8 inches long, and 3 feet wide at 2d. per square foot.

4. Find the cost of covering a box (except the bottom) with leather at 1s. per square foot, if the box measure 4 feet in length, 3 feet in breadth, and 3 feet in depth.

5. What will it cost to line the sides and bottom of a cistern $12\frac{3}{4}$ feet long, $8\frac{1}{4}$ feet wide, and $6\frac{1}{2}$ feet deep, with sheet lead 8 lbs. to the square foot, estimating the lead at 3d. per lb. ?

6. Find the cost of lining, at $4\frac{1}{2}$d. per square foot, a closed box whose external dimensions are 4 feet, 2 feet, and 16 inches respectively, the wood of which it is made being an inch thick.

CHAPTER IX

THE RIGHT PRISM AND THE RIGHT CYLINDER.

135. A parallelopiped has been defined (§ 127) as a solid, bounded by six parallelograms of which each opposite two are equal and parallel. It has also been stated that one pair of sides is usually called its ends.

136. Any solid whose two ends are similar, of equal area, and parallel to each other, is called a **prism**. The ends may be a triangle, square, pentagon, hexagon, etc.

137. According as its end or base is a triangle, square, pentagon, etc., a prism is called a triangular prism (A), a square prism (B), a pentagonal prism (C), etc.

A rectangular parallelopiped is hence one form of prism having its ends rectangles.

138. Since the ends are similar, equal, and parallel, the prism will have as many faces as there are sides in the base, each being a parallelogram.

139. When the side faces of a prism are perpendicular to the ends, it is called a **right prism.**

140. The perpendicular distance between the ends of a prism is called its *height.*

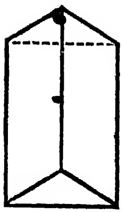


FIG. 41. (A)

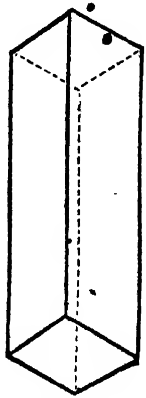


FIG. 42. (B)

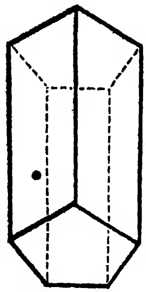


FIG. 43. (C)

141. To find the volume of a right prism.

Let it be required to find the volume of the triangular prism *ABD*, 7 inches long, the sides of each end being 5, 4 and 3 inches.

The solid may be considered as so many layers of the area of the base placed one over the other. Suppose we divide the solid *ABD* into 7 layers, each one inch long.

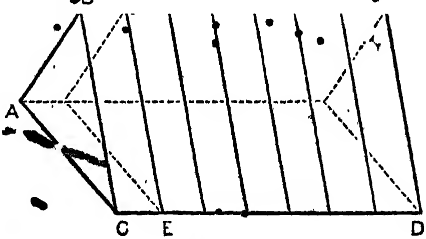


FIG. 44.

If *AB* be 3 inches, *AC* 4 inches, and *BC* 5 inches, the area of the end *ABC* will be found to be 6 square inches (§ 65); hence the solid *AEB* will be 6 cubic inches, and the volume of the whole prism will be 6 cubic inches $\times$ $7=42$ cubic inches; therefore **the volume of a prism is found by multiplying the area of the base by the length or height.**

2. The base of a prism is an equilateral triangle; each edge of the base measures 2 feet 8 inches, and the prism is 4 feet 6 inches long; find the area of the whole surface.

3. The base of a prism is an isosceles triangle, each of whose equal sides is 5 inches, and the third side 10 inches; its height is also 10 inches; find the area of its whole surface, in square inches.

4. Find the whole surface of a triangular prism whose height is 5 feet, and whose three sides of the base measure respectively 6 inches, 8 inches, and 10 inches.

5. The perpendicular height of a triangular stone pillar is 5 feet, and the edges of the base are 2 feet, 2 feet 6 inches, and 3 feet 6 inches respectively; what will it cost to paint the whole surface at 9d. per square foot?

6. The height of a square marble pillar is $7\frac{1}{2}$ feet; each edge of the base measures 2 feet 6 inches; find the cost of polishing all the sides at 2s. 3d. per square foot.

143. A solid whose two ends are equal and parallel circles is called a **cylinder.**

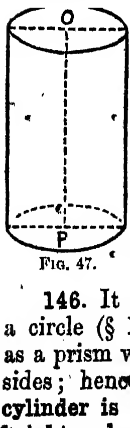


FIG. 47.

An ordinary round ruler and a stone roller are familiar examples of a solid cylinder.

144. The straight line *OP* between the two ends, from centre to centre, is called the *axis* of the cylinder.

145. When the axis *OP* is perpendicular to the ends, the cylinder is called a **right cylinder.**

The axis is the same as the height or length of the figure.

146. It has already been shown how a polygon may become a circle (§ 101). Now a cylinder may evidently be regarded as a prism whose base is a polygon of an indefinite number of sides; hence as in the case of the prism, **the volume of a cylinder is found by multiplying the area of the base by the height or length.**

Example.—Find the volume of a cylinder, the diameter of whose base is 21 inches, and whose height is 4 feet.

Here area of base $= 21 \times 21 \times \frac{11}{14} = 346\frac{1}{2}$ sq. inches (§ 102).

$\therefore$ Volume of cylinder $= (346\frac{1}{2} \times 48)$ cub. inches.
$= 16632$ cub. inches.
$= 9$ cub. feet 1080 cub. inches.

EXERCISE 99.

Find the volume of the following cylinders, when the dimensions given are respectively :—

1. Area of base, $38\frac{1}{2}$ sq. in. ; height, 4 in.
2. Diameter of base, 3 ft. 6 in. ; height, 16 ft.
3. Radius of base, 7 in. ; height, 10 in.
4. Circumference of base, 5 ft. 6 in. ; height, 20 ft.
5. Diameter of base, 2 ft. 4 in. ; height, 6 ft. 3 in.
6. Circumference of base, 3 ft. 8 in. ; height, 5 ft. 2 in.
7. Radius of base, 1 ft. 9 in. ; height, 1 ft. 9 in.
8. Diameter of base, 3 ft. 5 in. ; height, 11 ft. 9 in.
9. Radius of base, 1 ft. 2 in. ; height, 4 ft. 8 in.
10. Diameter of base, 7 ft. ; height, 10 ft. 6 in.

EXERCISE 100.

1. How many cubic yards must be dug out to make a round well 3 feet in diameter and 30 feet deep?

2. The diameter of a well is 3 feet 9 inches, and its depth 45 feet; find the cost of excavating it, at 9s. 4d. per cubic yard.

3. What would it cost to sink a cylindrical well 42 feet in depth, its diameter being $4\frac{1}{2}$ feet, at 7s. 4d. per cubic yard?

4. The top of a circular table is 7 feet in diameter and 1 inch thick; how many cubic feet and cubic inches of wood does it contain?

5. The roof of a hall is supported by 16 circular pillars of stone, each of which is $5\frac{1}{2}$ feet in circumference, and 20 feet high; find the cubical content of all the pillars.

6. A cheese has a diameter of 1 foot 2 inches and is 10 inches high; what is its solidity?

7. The trunk of a tree is 10 feet long, and it is a perfect cylinder; what is its cubic content, if the circumference is 7 feet 4 inches?

8. How many pieces of money, $\frac{3}{4}$ of an inch in diameter and $\frac{1}{8}$ of an inch thick, must be melted down in order to form a cube whose edge is 3 inches long?

9. The expense of excavating a circular well, of which the depth was 40 feet and the diameter 4 feet 1 inch, was £78 12s. 1d.; what was the charge per cubic yard?

10. A cylindrical boiler, whose diameter is 7 feet and length 10 feet, is full of water; find how many gallons it contains, reckoning $6\frac{1}{4}$ gallons to the cubic foot.

147. To find the number of cubic feet (or cubic inches) in a cylindrical shell or pipe.

Drain pipes, gas pipes, chimney pots, and the ordinary garden-roller, are familiar illustrations of a hollow cylinder.

Example.—The exterior diameter of a cast iron pipe is 7 inches, the thickness of the metal $\frac{1}{2}$ inch, and the length of the pipe 14 feet; find the number of cubic inches of iron in it.

The interior diameter $= 7$ inches $- (\frac{1}{2}$ inch $\times 2) = 6$ inches.

Then area of larger circle $= \left(7 \times 7 \times \frac{11}{14}\right)$ sq. inches $= 38\frac{1}{2}$ sq. inches.

And area of smaller circle $= \left(6 \times 6 \times \frac{11}{14}\right)$ sq. inches $= 28\frac{2}{7}$ sq. inches.

$\therefore$ Area of the ring $= (38\frac{1}{2} - 28\frac{2}{7})$ sq. inches $= 10\frac{3}{14}$ sq. inches (§ 106)

$\therefore$ Volume of the pipe $= (10\frac{3}{14} \times 168)$ cub. in. $= 1716$ cub. in. (§ 146)

EXERCISE 101.

1. The outer diameter of a metal pipe is 12 inches, the inner diameter is 10 inches, and the length of the pipe is 20 feet; find the number of cubic feet of metal in it.

2. What is the volume of a hollow cylinder whose inner and outer diameters are 14 inches and $17\frac{1}{2}$ inches, if the cylinder be 4 feet high?

3. Find the volume of a hollow cylinder, the exterior circumference of which is 5 feet 6 inches and the internal circumference 3 feet 8 inches, the length of the cylinder being 5 feet 6 inches.

4. The exterior diameter of a hollow iron roller is 4 feet 8 inches, the thickness of the iron $3\frac{1}{2}$ inches and the length of the roller 6 feet; find its solid content.

NOTE.—The inner diameter = outer diameter − twice the thickness of the metal.

5. What is the volume of a cylindrical pipe 14 inches long, the diameter of the inner surface being 6 inches, and the thickness of the metal 1 inch?

NOTE.—The outer diameter = inner diameter + twice the thickness of the metal.

6. Find the volume of a cylindrical shell, the radius of the inner surface being $16\frac{1}{2}$ inches, the thickness 2 inches, and the height 12 feet.

148. The volume of a cylinder and the area of the base (or the height) being given, the height (or dimensions of the base) can be readily found.

Example 1.—The volume of a cylinder is 9 cub. feet 1080 cub. inches, and the diameter of its base is 1 ft. 9 in.; find the height.

Since volume = area of base × height;

$$\therefore \text{Height} = \frac{\text{volume}}{\text{area of base}}.$$

Here volume = 16632 cub. inches and area of base $= \left(21 \times 21 \times \frac{11}{14}\right)$ square inches $= \frac{693}{2}$ square inches.

$$\therefore \text{Height} = \frac{16632}{\frac{693}{2}} = 48 \text{ inches} = 4 \text{ feet.}$$

Example 2.—Find the diameter of the base of a cylinder whose volume is 3 cub. feet 360 cub. in. and whose height is 3 feet.

$$\text{Area of base} = \frac{\text{volume}}{\text{height}}.$$

Here volume = 5544 cubic inches and height = 36 inches.

$$\therefore \text{ Area of base} = \frac{5544}{36} \text{ square inches} = 154 \text{ square inches.}$$

Since area of circular base = 154 sq. inches

and diameter = square root of $\left(\text{area} \div \frac{11}{14}\right)$; (§ 104)

$$\therefore \text{ Diameter} = \sqrt{\frac{154 \times 14}{11}}.$$
$$= \sqrt{14 \times 14}.$$
$$= 14 \text{ inches.}$$

EXERCISE 102

1. The volume of a cylinder is 126 cubic feet and the area of the base $10\frac{1}{2}$ square feet; what is its length?

2. The cubic content of a cylinder is 19 cubic feet 168 cubic inches and its height 5 feet 6 inches; what is the area of its base?

3. The circular base of a cylinder has a diameter measuring 2 feet 4 inches and its cubic content is 21 cubic feet 672 cubic inches; what is the height of the cylinder?

4. The cubic content of a cylinder is 9 cubic feet 1080 cubic inches, and its height is 4 feet; what is the radius of its circular base?

5. A cylinder is 3 feet 6 inches long and contains 2 cubic yards 5 cubic feet 1536 cubic inches; find the diameter of its circular end.

6. The height of a cylinder is $6\frac{1}{2}$ feet, and its cubic content 4 cubic yards 3 cubic feet 384 cubic inches; find the circumference of its circular base.

7. A cubic foot of brass is made into a wire $\frac{1}{11}$ of an inch in diameter; how many yards long is the wire?

8. If 114 cubic yards 2 cubic feet of earth are dug out to make a well 14 feet in diameter, find its depth.

9. The solid content of a cylindrical iron rod is 15 cubic feet 54 cubic inches, and it is 35 feet long; find the diameter of its circular end.

10. Three cubic feet are to be cut off from a cylindrical roller 44 inches in circumference; how far from the end must the section be made?

149. To find the whole surface of a cylinder.

Let *ABCD* be a hollow cylinder of cardboard or paper, and suppose it to be cut open along the end *EF*, and the surface then laid out flat. It is evident it will take the form of the rectangle *EFF′E′*, whose length *FF′* is equal to the circumference

of the base of the cylinder, and breadth EF equal to the height of the cylinder. Now the area of the rectangle is the product

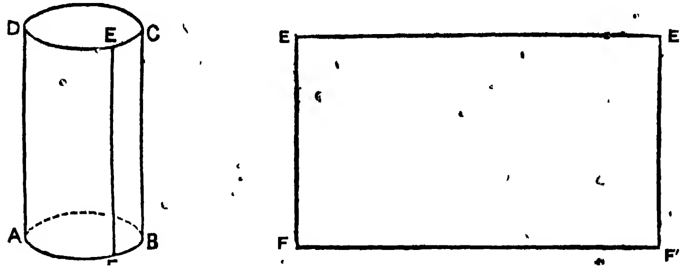


FIG. 48.

of the length and breadth; therefore **the area of the curved surface of a right cylinder is the product of the circumference of the base and the height or length.**

The whole surface of a cylinder is the area of the ends together with the area of the curved surface.

Example.—The diameter of the base of a right cylinder is 14 inches and the height is 10 inches; find the area of the curved surface, and of the whole surface.

Circumference of base $= \left(14 \times \frac{22}{7}\right)$ inches $= 44$ inches.

$\therefore$ Area of curved surface $= (44 \times 10)$ sq. inches $= 440$ sq. inches
$= 3$ sq. ft. 8 sq. in.

Area of the two ends $= \left(14 \times 14 \times 2 \times \frac{11}{14}\right)$ sq. inches $= 308$ sq. in.
$= 2$ sq. ft. 20 sq. in.

$\therefore$ Area of whole surface $= 3$ sq. ft. 8 sq. in. $+ 2$ sq. ft. 20 sq. in.
$= 5$ sq. ft. 28 sq. in.

EXERCISE 103.

Find the area of the curved surface of right cylinders with the following dimensions:—

1. Circumference of base, 2 ft. 11 in.; height, 6 ft.
2. Diameter of base, 4 ft. 8 in.; height, 7 ft. 6 in.
3. Radius of base, 1 ft. 2 in.; height, 5 ft.

Find the area of the whole surface of right cylinders with the following dimensions :—

4. Circumference of base, 14 ft. 8 in. ; height, 8 ft. 6 in.
5. Diameter of base, 5 ft. 10 in. ; height, 9 ft.
6. Radius of base, 1 ft. 2 in. ; height, 7 ft. 4 in.

EXERCISE 104.

1. Find the cost of painting the curved surfaces of 5 cylindrical pillars, each 14 feet high and 1 foot in diameter, at 8½d. per square yard.

2. What is the expense of polishing the curved surface of a marble column, 7 feet 4 inches in circumference and 15 feet high, at 3s. 4d. per square foot?

3. What is the expense of painting the walls of a cylindrical room 16 feet high and 18 feet in diameter, at 7½d. per square yard?

4. A cylindrical cask, closed at both ends, is 3 feet high and 3 feet in diameter; what will it cost if the price of the wood be 6d. per square foot?

5. The diameter of the end of an iron roller is 9 feet 4 inches, and it is 12 feet long; what space will it roll in one complete turn?

6. How many revolutions of a roller, 3 feet in length, and 18 inches in diameter, would it take to go over a grass-plot half an acre in extent?

7. The diameter of a stone roller is 1 foot 9 inches, and it is 3 feet long; it makes 1320 revolutions in passing from one end of a lawn to the other; find the area rolled.

8. The circumference of a roller is 11 feet, and it is 6 feet long; it makes 30 revolutions in passing from one end of a cricket ground to the other; find the area of the ground, if it is rolled in 22 journeys.

CHAPTER X.

THE RIGHT PYRAMID AND THE RIGHT CONE.

150. A solid whose base is any plane rectilineal figure, such as a triangle, square, polygon, etc., and whose side-faces are three or more triangles, all of which meet in one common point, is called a **pyramid**.

151. According as its base is a triangle, square, hexagon, etc., a pyramid is called a triangular pyramid, a square pyramid, a hexagonal pyramid, etc.

152. The common point in which all the sides terminate is called the *vertex* of the pyramid.

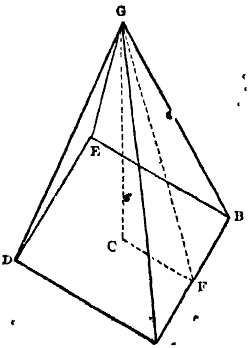


FIG. 49.

153. When the straight line GC, drawn from the vertex G to the centre C of the base is perpendicular to the base, the pyramid is called a **right pyramid.**

154. GC is the *perpendicular height* of the pyramid, or, as it is simply called, the *height.*

155. The line drawn from the vertex to the middle point of any one of the sides of the base, as GF, is called the *slant height* of the pyramid.

156. GA, GB, GE and GD are called the *edges* of the pyramid, and all are equal.

157. To find the volume of a right pyramid.

It has been proved by mathematicians that the volume of any pyramid is one-third of that of a prism having the same base and height.

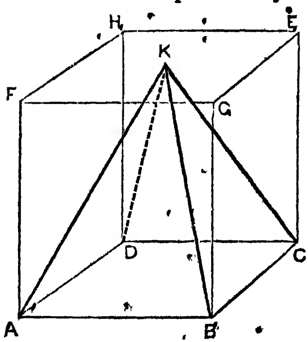


FIG. 50.

Let $ABCDEFGH$ be a cubical vessel, *i.e.* a rectangular prism, and suppose it to be filled with three pints of water. Place carefully into it a pyramid having the same base and height; it will be found that one pint, *i.e.* one-third of the water in the prism will overflow. Now the volume of a prism is the product of the area of the base by the height; hence **the volume of a pyramid is found by multiplying the area of the base by the perpendicular height and taking one-third of the product.**

Example.—The base of a pyramid is a square, each side of which is 18 feet 6 inches; the perpendicular height is 30 feet; find the volume.

Here area of base $= (18\frac{1}{2} \times 18\frac{1}{2})$ sq. feet.

$$\text{Volume of pyramid} = \frac{18\frac{1}{2} \times 18\frac{1}{2} \times 30}{3} \text{ cub. feet}$$
$$= 3422\tfrac{1}{2} \text{ cub. feet}$$
$$= 3422 \text{ cub. ft. } 864 \text{ cub. in.}$$

NOTE.—When the *height* of a pyramid is mentioned it always refers to *perpendicular height.*

EXERCISE 105.

Find the volume of the following pyramids, when the dimensions given are respectively—

1. Base, square; side, 30 ft.; height, 25 ft.
2. Base, square; side, 5 ft. 8 in.; height, 27 ft.
3. Base, square; side, 2 ft. 9 in.; height, 5 ft. 3 in.
4. Base, square; side, $3\frac{1}{2}$ft.; height, $7\frac{1}{2}$ ft.
5. Base, scalene triangle; sides, 3 ft., 4 ft., 5 ft; height, 6 ft.
6. Base, equilateral triangle; side, 5 ft.; height, 30 ft.
7. Base, scalene triangle; sides, 25 in., 29 in., 36 in.; height, $14\frac{2}{5}$ in.
8. Base, equilateral triangle; side, 6 ft.; height, 28 ft.

EXERCISE 106.

1. The length of the line drawn from the vertex of a pyramid to the centre of its square base, each side of which measures $3\frac{1}{2}$ feet, is $5\frac{1}{2}$ feet; find its volume.
2. Find the weight in tons, etc., of a square pyramid of stone 50 feet high, each side of the base being 3 feet 6 inches, and a cubic foot of stone weighing 240 lbs.
3. Find the cost of a triangular piece of wood at $7\frac{1}{2}$d. per solid foot, the sides of the base being 13, 14, and 15 feet respectively, and the perpendicular height 63 feet.
4. The largest of the Egyptian pyramids is 482 feet high, and its base is 758 feet square; find the volume in cubic yards, etc.
5. A vessel in the form of a square pyramid is 6 feet 6 inches deep, and each side of its base is 3 feet 6 inches; how many cubic feet and cubic inches of water will it hold?
6. Find the weight in tons, etc., of a triangular pyramid of marble, 10 feet high, the sides of which measure 5, 4, and 3 feet respectively, the weight of the marble being 180 lbs. per cubic foot.

158. To find the whole surface of a right pyramid.

Let $ABCD$ be a triangular pyramid. It is evident that the total area of the side faces will equal the area of the sides ADB, BDC and ADC.

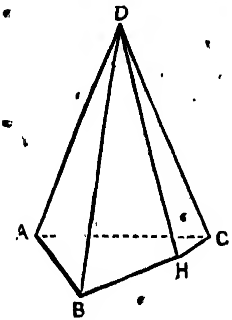


FIG. 51.

Draw DH perpendicular to BC. This will be the slant height of the pyramid.

Now area of face $ADB = \frac{AB \times DH}{2}$

and area of face $BDC = \frac{BC \times DH}{2}$

and area of face $ADC = \frac{AC \times DH}{2}$

so that the total surface of the sides $= (AB + BC + AC) \times \frac{DH}{2}$; but $AB + BC + AC$ is the perimeter of the base and $\frac{DH}{2}$ is one-half the slant height; therefore, **the area of the side faces of a pyramid is found by multiplying the perimeter of the base by one-half the slant height.**

The whole surface of a pyramid is the area of the base together with the area of the side faces.

NOTE.—Bear in mind that it is necessary to know the *slant* height in order to find the area of the side faces.

Example 1.—A square pyramid measures 10 feet along each side of the base and its slant height is 24 feet. Find the area of the triangular faces, and of the whole surface.

Here perimeter of base $= 10$ feet $\times 4 = 40$ feet;

$\therefore$ Area of triangular faces $= \frac{40 \times 24}{2} = 480$ sq. feet

Now area of base $= (10 \times 10)$ sq. feet $= 100$ square feet;

$\therefore$ Area of whole surface $= 480$ sq. ft. $+ 100$ sq. ft. $= 580$ sq. feet.

Example 2.—The perpendicular height of a square pyramid is 35 feet; each side of the base is 24 feet; find the area of the side faces.

Referring to figure 49, we have the right-angled triangle GCF, where GC is 35 feet, and CF is 12 feet (half the length of a side of the base); hence it is necessary to find, in the first place, the slant height GF, which is equal to $\sqrt{35^2 + 12^2} = \sqrt{1369} = 37$ feet.

Here perimeter of base $= 24$ feet $\times 4 = 96$ feet.

$\therefore$ Area of side faces $= \frac{96 \times 37}{2} = 1776$ square feet.

EXERCISE 107.

1. Find the area of the side faces of a pyramid on a square base; each side of the base being 18 feet and the slant height 30 feet.

2. A pyramid whose slant height is 2 feet 9 inches has for its base an equilateral triangle, each side of which is 2 feet 6 inches; find the area of the side faces.

3. Find the whole surface of a square pyramid, each side of the base of which is 12 feet, and its slant height 25 feet.

4. A pyramid stands on a square base which is 2 feet 8 inches long; its slant height is 4 feet; find the whole surface.

5. In a square pyramid the side of the base is 3 feet 6 inches and the slant height 5 feet 4 inches; find the whole surface.

6. Find the whole surface of a triangular pyramid each side of whose base is 6 feet, the slant height being 18 feet.

7. The slant height of a triangular pyramid is 20 feet, and each side of the base is 8 feet 4 inches long; find the whole surface.

8. The perpendicular height of a square pyramid is 24 feet, each side of the base being 20 feet; find the area of the side faces.

9. It is desired to cover a piece of ground 80 feet square by a pyramidal tent 30 feet in perpendicular height; find the cost of the necessary quantity of canvas at $4\frac{1}{2}$d. per square yard.

159. A solid whose base is a circle, and whose curved surface tapers to a point or *vertex* is called a **cone**.

160. When the straight line DC drawn from the vertex D to the centre C of the circular base is perpendicular to the base, the cone is called **a right cone**.

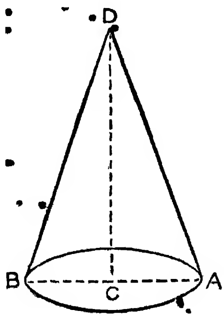


FIG. 52.

161. DC is the *perpendicular height* of the cone, which is necessary to be known before finding the volume. DA or DB is the *slant height*, which is necessary to be known before finding the curved surface.

162. To find the volume of a cone.

It has been proved by mathematicians that pyramids and cones of equal bases and altitudes are equal to one another. The volume of a cone may therefore be obtained from that of the pyramid; for a cone may be regarded as a pyramid having an indefinite number of sides. Therefore the perimeter of the pyramid ultimately becomes the circumference

of the cone; hence, as in the case of the pyramid, **the volume of a cone is found by multiplying the area of the base by the perpendicular height, and taking one-third of the product.**

Example 1.—The diameter of the base of a right cone is 7 feet, and the perpendicular height 10 feet; required the volume.

$$\text{Here area of base} = 7 \times 7 \times \frac{11}{14} = 38\tfrac{1}{2} \text{ sq. ft. (§102)}$$

$$\therefore \text{Volume of cone} = \frac{38\frac{1}{2} \times 10}{3} = 128\tfrac{1}{3} \text{ cub. ft.}$$

$$= 128 \text{ cub. ft. } 576 \text{ cub. in.}$$

Example 2.—The radius of the base of a right cone is 12 feet, and the slant height 37 feet; required the volume.

Referring to figure 52, we have the right-angled triangle ACD, where DA is 37 feet and AC is 12 feet; hence it is necessary to find, in the first place, the perpendicular height DC, which is equal to $\sqrt{37^2 - 12^2} = \sqrt{1225} = 35$ ft.

$$\text{Here area of base} = 24 \times 24 \times \frac{11}{14} = 452\tfrac{4}{7} \text{ sq. ft.}$$

$$\therefore \text{Volume of cone} = \frac{452\frac{4}{7} \times 35}{3} = 5280 \text{ cub. ft.}$$

EXERCISE 108.

Find the volume of the following cones, when the dimensions given are respectively :—

1. Area of base, 4 sq. ft. 40 sq. in.; perpendicular height, 1 ft. 3 in.
2. Diameter of base, 1 ft. 2 in.; perpendicular height, 7 ft.
3. Radius of base, 1 ft. 9 in.; perpendicular height, 7 ft. 6 in.
4. Circumference of base, 77 ft.; perpendicular height, 16 ft.
5. Diameter of base, 5 ft. 10 in.; perpendicular height, 5 ft. 6 in.
6. Radius of base, 2 ft. 4 in.; perpendicular height, 9 ft.
7. Circumference of base, 22 ft.; perpendicular height, 7 ft. 3 in.
8. Circumference of base, 44 ft.; slant height, 25 ft.
9. Radius of base, 3 ft. 6 in.; slant height, 5 ft. 10 in.
10. Diameter of base, 4 ft. 8 in.; slant height, 4 ft. 5 in.

EXERCISE 109.

1. What is the volume of a sugar-loaf which measures 44 inches all round the base and is 15 inches high?
2. What quantity of water is contained in a conical vessel whose depth is 10 feet, its circular top having a diameter of 7 feet?
3. How many cubic feet of air are there in a conical-shaped tent 8 feet high, and 10 feet in diameter?
4. A conical wine-glass is 2 inches wide at the top, and 3 inches deep; how many cubic inches of wine will it hold?

5. A conical mound of earth is 264 yards in circumference, and the length of its slope, from the foot to its summit, is 70 yards; how many cubic yards of earth are in the mound?

6. How many gallons of water are contained in a vessel, which is in the shape of a right cone, the radius of the base being 6 feet, and the length of the slant height 10 feet, assuming that a gallon occupies $277\frac{1}{4}$ cubic inches?

163. To find the whole surface of a right cone.

Let *ABC* be a hollow cone of cardboard or paper, and suppose it to be cut open along the line *AB*, and the surface then laid out flat.

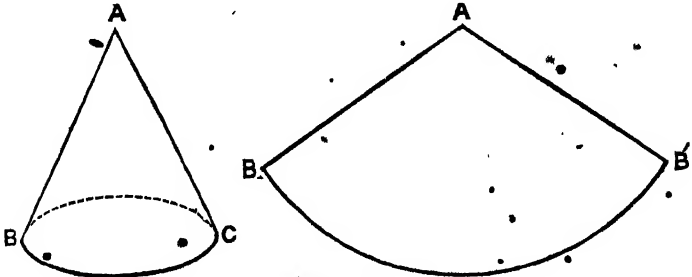


Fig. 53.

It is evident it will take the form of a sector of a circle *ABB′*, the radius of the sector being the slant height of the cone, and the length of the arc the circumference of the cone. Now the area of a sector of a circle is half the product of the length of the arc and the radius (§ 115); therefore **the area of the curved surface of a cone is half the product of the circumference of the base and the slant height.**

The whole surface of a cone is the area of its circular base together with the area of the curved surface.

Example 1.—The diameter of the base of a cone is 2 feet 4 inches and the slant height is 2 feet 2 inches; find the area of the curved surface, and of the whole surface.

Here circumference of base $= 2\frac{1}{3}$ feet $\times \frac{22}{7} = 7\frac{1}{3}$ ft.

$\therefore$ Curved surface $= \dfrac{7\frac{1}{3} \times 2\frac{1}{6}}{2} = 7$ sq. ft. 136 sq. in.

Area of base $= \left(2\frac{1}{3} \times 2\frac{1}{3} \times \frac{11}{14}\right)$ sq. ft. $= 4$ sq. ft. 40 sq. in.

$\therefore$ Whole surface $= 7$ sq. ft. 136 sq. in. $+ 4$ sq. ft. 40 sq. in.
$= 12$ sq. ft. 32 sq. in.

Example 2.—Find the curved surface of a cone whose radius is 14 inches and perpendicular height 48 inches.

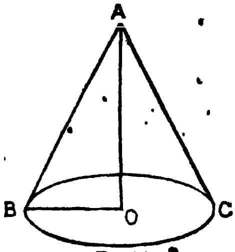


FIG. 54.

The perpendicular height AO of the cone, the radius BO of the base and the slant height AB form respectively the height, base, and hypothenuse of a right-angled triangle.

Here we have given the height 48 inches and the base 14 inches; hence the hypothenuse is equal to $\sqrt{48^2+14^2}=\sqrt{2500}=50$ inches, *i.e.* the the slant height of the cone is 50 inches.

Since radius = 14 inches; diameter = 28 inches.

$\therefore$ Circumference of base $= 28 \text{ inches} \times \frac{22}{7} = 88$ inches.

$\therefore$ Curved surface = (88 × 50) sq. inches.
= 4400 sq. inches.
= 30 sq. feet 80 sq. inches.

EXERCISE 110.

Find the area of the curved surface of right cones with the following dimensions :—

1. Circumference of base, 44 ft.; slant height 25 ft.
2. Diameter of base, 3 ft. 6 in.; slant height 6 ft. 3 in.
3. Radius of base, 2 ft. 4 in.; slant height 4 ft. 5 in.
4. Diameter of base, 1 ft. 9 in.; slant height 2 ft. 8 in.
5. Radius of base, 2 ft. $7\frac{1}{2}$ in.; slant height 12 ft.
6. Circumference of base, 7 ft. 4 in.; slant height 10 ft.
7. Circumference of base, 7 ft. 4 in.; perpendicular height 4 ft.
8. Diameter of base, 11 ft.; perpendicular height 30 ft.
9. Radius of base, $24\frac{1}{2}$ ft.; perpendicular height 84 ft.
10. Radius of base, 6 ft.; perpendicular height 8 ft.

EXERCISE 111.

1. What will be the cost of painting a conical spire whose slant height is 120 feet, and whose circumference at the base is 72 feet, at $8\frac{1}{2}$d. per square yard?

2. How many cubic feet of lead will be required to cover a conical spire which measures 35 feet in circumference and whose slant height is 30 feet; supposing that the lead is $\frac{1}{12}$ of an inch thick?

3. The slant height of a conical-shaped tent is 12 feet and the tent occupies $38\frac{1}{2}$ square feet; how many square yards of canvas were required?

4. How many square yards of canvas will be required to form a conical-shaped tent, the height of the central pole being 10 feet and the diameter of the tent 19 feet 10 inches?

5. What will be the cost of canvas for a conical tent 8 feet high, and 12 feet in diameter, at 1s. $3\frac{3}{4}$d. per square yard.

6. How many yards of canvas, $\frac{3}{4}$ yards wide, will be required for a conical tent 10 feet high and 16 yards in diameter?

CHAPTER XI.

THE SPHERE.

164. A solid bounded by a curved surface every point of which is equally distant from its middle point or centre is called a **sphere**.

A sphere is sometimes called a globe; marbles and billiard balls are familiar examples of a sphere.

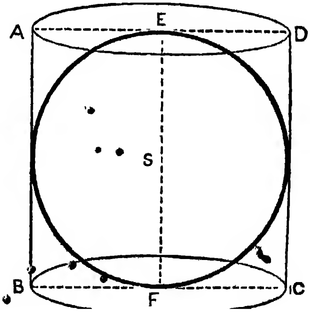
165. The straight line drawn from the centre to the surface is the *radius* of the sphere.

166. The straight line drawn through the centre and terminated at both ends by the surface is the *diameter* of the sphere.

167. To find the volume of a sphere.

It has been proved by mathematicians that the volume of a sphere is two-thirds of the volume of that of a cylinder having the same diameter and height.

Let *ABCD* be a cylindrical vessel whose height *EF* is equal to the diameter *BC* of the base. Suppose it to be filled with three pints of water. Place carefully into it a sphere *S* having the same diameter and the same height; it will be found that two pints, *i.e.* two-thirds of the water in the cylinder will overflow. Now the volume of a cylinder is the product of the area of the base by the height; therefore since the length of the diameter is given, the volume of the cylinder *ABCD* is equal to $\frac{11}{14}\times$ diameter $\times$ diameter $\times$ height (§ 146), and since the height of the cylinder *ABCD* and the diameter of its base are equal, the volume of the cylinder *ABCD* may be written as $\frac{11}{14}\times$ cube of the diameter and the



volume of the sphere S as $\frac{2}{3} \times \frac{11}{14} \times$ cube of the diameter, or $\frac{11}{21} \times$ cube of the diameter; hence **the volume of a sphere is found by multiplying the cube of the diameter by $\frac{11}{21}$.**

Example.—What is the volume of a sphere whose diameter is 3 feet 6 inches?

Here, 3 ft. 6 in. $= 42$ in.

$\therefore$ Volume of sphere $= 42 \times 42 \times 42 \times \frac{11}{21} = 38808$ cub in.

$= 22$ cub. ft. 792 cub. in.

EXERCISE 112.

Find the volume of the following spheres, whose diameters are :—

1. 7 ft. 2. 3 ft. 6 in. 3. 5 ft. 3 in. 4. 2 ft. 11 in.

Find the volume of the following spheres, whose radii are :—

5. 7 in. 6. 5 ft. 3 in. 7. 8 ft. 9 in. 8. 1 ft.

Find the volume of the following spheres, whose circumferences are :—

9. 7 ft. 4 in. 10. 5 ft. 6 in. 11. 27 ft. 6 in. 12. 20 ft. 2 in.

EXERCISE 113.

1. How many cubic inches of iron are there in a spherical cannon ball, 9 inches in diameter?

2. The weight of a cubic inch of iron is $4\frac{1}{2}$ ounces, avoirdupois; how much will a solid ball weigh whose diameter is 5 inches?

3. Find in cwt. and lbs. the weight of 21 cast-iron cannon balls each 4 inches in diameter, supposing that a cubic inch of iron weighs $4\frac{1}{2}$ ounces.

4. How many gallons of water can be contained in a hemispherical vessel 2 feet 4 inches in diameter, supposing that a gallon occupies 277 cubic inches?

5. Find in lbs. the weight of a hemispherical block of stone 2 yards in radius, supposing 4 cubic feet of stone to weigh 9 cwt.

6. Find the weight in tons, etc., of the water in a hemispherical vessel $10\frac{1}{2}$ feet in radius, supposing that a cubic foot of water weighs 1000 ounces.

168. To find the surface of a sphere.

It has been proved by mathematicians that the surface of a sphere is equal to the curved surface of a cylinder of the same diameter and height. Now the curved surface of a cylinder is found by multiplying its circumference by its perpendicular height (§ 149), which may be written $\frac{22}{7} \times$ diameter $\times$ height,

and since the height of the cylinder and the diameter of its base are in this instance equal, this may be briefly written $\frac{22}{7}\times$ square of the diameter; hence **the surface of a sphere is found by multiplying the square of the diameter by $\frac{22}{7}$.**

Example.—How many square inches of gold-leaf will gild a globe 7 inches in diameter?

$$\text{Surface of sphere} = \left(7\times 7\times \frac{22}{7}\right) \text{ sq. inches.}$$
$$= 154 \text{ sq. inches.}$$

EXERCISE 114.

Find the surface of the following spheres whose diameters are :—

1. 56 ft. 2. 1 ft. 2 in. 3. 11 ft. 8 in. 4. 2 ft. 11 in.

Find the surface of the following spheres whose radii are :—

5. 7 ft. 6. 1 ft. 2 in. 7. 1 ft. 9 in. 8. 5 ft. 3 in.

EXERCISE 115.

1. How many square inches of gold-leaf will gild a ball 18 inches in diameter?
2. How many square feet and square inches of cloth will cover a ball 1 foot $5\frac{1}{2}$ inches in diameter?
3. Find the cost of enamelling a spherical ball 6 inches in diameter, at 5s. 3d. per square inch.
4. What will be the cost of gilding a ball $1\frac{3}{4}$ feet in radius, at 6s. per square foot?
5. Supposing the ball on the top of St. Paul's Cathedral to be a perfect sphere, 6 feet in diameter, what would be the cost of gilding it at $3\frac{1}{2}$d. per square inch?
6. The circumference on the dome of a large building, in the shape of a hemisphere, is 66 feet; how many square feet of lead will be necessary to cover it?

169. To find the volume of a spherical shell.

It is evident the volume of a spherical shell may be determined by subtracting the volume of the hollow part considered as a solid from the volume of the whole sphere considered as a solid; hence **the volume of a spherical shell is found by subtracting the cube of the inner diameter from the cube of the outer diameter and multiplying the difference by $\frac{11}{21}$.**

NOTE.—1. The outer diameter = the inner diameter + twice the thickness of the shell.

2. The inner diameter = the outer diameter − twice the thickness of the shell.

Example.—The outer diameter of a spherical shell is 18 inches, and the thickness of the shell $4\frac{1}{2}$ inches; find the number of cubic inches in the volume.

Here inner diameter = 18 inches − 9 inches = 9 inches.

$$\therefore \text{Volume of shell} = (18^3 - 9^3) \text{ inches} \times \frac{11}{21}$$
$$= (5832 - 729) \text{ inches} \times \frac{11}{21}$$
$$= 5103 \text{ inches} \times \frac{11}{21}$$
$$= 2673 \text{ cubic inches.}$$

EXERCISE 116.

1. Find the solid content of a spherical shell whose external and internal diameters are respectively 1 foot 7 inches and 1 foot 5 inches.

2. The exterior diameter of a hollow sphere is 22 inches, and the thickness of the shell 3 inches; find its solidity.

3. Find the number of cubic inches of metal in a hollow shell, whose interior diameter is 10 inches, and whose thickness is one inch.

4. The inside diameter of a hollow sphere of metal is 18 inches and the thickness 2 inches; what is its weight, supposing that a cubic foot of metal weighs 7776 ounces?

5. Find the weight in lbs. of a bombshell whose exterior and interior diameters are respectively 10 and 8 inches, assuming that a cubic foot of iron weighs 7210 ounces.

6. What is the weight of a spherical shell 11 inches in diameter and 3 inches thick, composed of iron weighing 4 cwt. to the cubic foot?

EXERCISE 117.

Revision Examination.—Solids.

1. How many cubic inches of iron will be required to make a garden roller which is $\frac{1}{2}$ inch thick, with an outer circumference of $5\frac{1}{2}$ feet, and a length of $3\frac{1}{2}$ feet?

2. A pyramid of lead is 14 inches high, and stands on a square base 6 inches long on each side; how many spherical bullets $\frac{3}{4}$ inch in diameter can be made out of it?

3. A cubic foot of copper is drawn into a wire $\frac{1}{10}$ of an inch in diameter; how many feet long is the wire?

4. A shell is 7 inches in external diameter and 2 inches thick; how many ounces of powder will fill it, if 30 cubic inches of powder weigh 1 lb.?

5. A circular shaft is 90 feet deep and 3 feet 9 inches in diameter; find the cost of sinking it at 14s. 6d. per cubic yard.

6. Find, in cubic feet, the volume of a cone, the radius of whose base is 4 feet 6 inches and whose height is equal to its circumference.

7. A mound of earth in the shape of a pyramid occupies 185 square feet of ground, and is 45 feet in perpendicular height; find the number of cubic yards in the mound.

8. A cylindrical leaden pipe is 28 feet long, $2\frac{1}{2}$ inches in exterior diameter and $2\frac{1}{8}$ inches in interior diameter; how many spherical bullets, each 6 inches in diameter, can be made out of it?

9. Find the weight, in cwt., etc., of 8 fir planks, each 6 feet 6 inches long, $11\frac{1}{4}$ inches wide, and 4 inches thick, supposing a cubic foot of fir to weigh 33 lbs.

10. A canal, 4 miles long, is 30 feet wide at the top, 18 feet wide at the bottom, and 6 feet deep; how many cubic yards of earth were excavated in digging it?

11. The diameter of the base of a cone is 9 feet, and the height is equal to the circumference of the base; find its value at 1s. 6d. per cubic foot.

12. How many spherical bullets, each $\frac{1}{2}$-inch in diameter, could be cast from a block of lead 1 foot 10 inches long, 1 foot 2 inches wide and 5 inches thick?

EXERCISE 118.

Examination Tests.—Circles and Solids.

A.

1. Find the number of bricks required to build a wall 26 yards long, 7 feet 8 inches high, and 14 inches thick, each brick and its mortar being 9·1 inches long, 4·6 inches broad, and 3·2 inches deep.

2. At 2s. 6d. a square foot, what is the cost of polishing the convex surface of a cylinder $10\frac{1}{2}$ inches in diameter and 8 feet long?

3. The area of the base of a triangular pyramid is 24 square feet, and its perpendicular height is 50 feet; find its volume.

B.

4. A closed vessel made of plank $1\frac{1}{2}$ inches thick is externally 1 yard long, 1 yard deep, and 1 yard broad. Find the cost of lining it with a thin metal covering at 4d. a square foot.

5. A cubic inch of brass is drawn into a wire $\frac{1}{25}$ of an inch diameter; find the length of the wire to the nearest inch.

6. Find the volume of a triangular right prism whose length is 5 feet, and the sides of whose base are 6 inches, 8 inches, and 10 inches respectively.

C.

7. Find by duodecimals the solid content of a block of marble 3 ft. 8′ 4″ long, 1 ft. 7′ 6″ broad, and 1 ft. 4′ 9″ thick. Express your result in cubic feet and inches.

8. How many gallons of water will be required to fill a cylindrical tank 20 feet 2 inches deep and 21 feet in diameter?

9. The slant height of a cone being 50 feet and the diameter of the base $8\frac{1}{2}$ feet; find the area of the convex surface.

D.

10. What is the value of a log of wood 18 feet long, 3 feet 2 inches broad, and 2 feet thick, at 7s. 6d. per cubic foot?

11. The height of a cone is 1 foot 8 inches, the circumference at the base 6 feet; find its volume in cubic feet.

12. How many cubic yards, etc. were dug out in making a well whose depth was 15 feet, and diameter 3 feet 6 inches?

E.

13. How many revolutions a minute would a wheel 176 inches in circumference have to make in order to travel at the rate of 15 miles an hour?

14. Find the surface of a triangular prism whose length is 5 feet, and the sides of whose base are 6 inches, 8 inches, and 10 inches respectively.

15. How many cubic inches of metal will be required to make a hollow spherical ball, the external diameter being 18 inches, and the thickness $4\frac{1}{2}$ inches?

F.

16. A brick is 9 inches long, $4\frac{1}{2}$ inches wide, and 3 inches thick, and weighs 6 lbs. How many bricks would be required to build a wall 1 mile long, 7 feet high, and 18 inches thick, and how many tons would it weigh?

17. A cone of silver whose base is 18 inches in diameter and whose height is $4\frac{1}{2}$ inches, is melted and made into a spherical ball; find the diameter of the ball.

18. Find the total surface of a right cylinder whose diameter is 21 feet, and height 2 feet 4 inches.

G.

19. Find the cost of plating a cube of metal containing 2 cubic feet 1457 cubic inches with silver at 3s. 9d. per square foot.

20. Find the total surface of a square pyramid whose edge measures 3 feet 6 inches; and the slant height 5 feet 4 inches.

21. The diameters of the wheels of a bicycle are 48 and 14 inches respectively; find how many more revolutions the small wheel will make than the large wheel in a distance of 10 miles.

H.

22. How many square feet of metal will be required to make a rectangular tank, open at the top, 12 feet long, 10 feet broad, and 8 feet

23. The area of a circle is $38\frac{1}{2}$ square feet; find the side of a square whose perimeter is equal to the circumference of the circle.

24. What is the volume of a triangular pyramid 5 feet high, if the sides of the base measure respectively 1 foot 3 inches, 1 foot 8 inches and 2 feet 1 inch?

CHAPTER XII.

EXAMINATION PAPERS.

PAPER 1.

1. The sides of a triangular portion of a field are 25, 113, 132 yards; and this portion is an eleventh part of the whole field; how many acres does the field contain?

2. The diameters of two circles are 2·58 and 3·65 feet; by how much does the area of the greater exceed that of the less?

3. Find, to three decimal places, the side of a square equal in area to a trapezoid of which the width is 9 inches, and the parallel sides are 26 and 37 inches.

4. What is the area of an equilateral triangle, its side being 26 inches?

PAPER 2.

1. The sides of a triangle are 3, 5, 7 feet respectively; find its area.

2. The length of a rectangular field is 1400 links, and its diagonal measure is 1490 links; find the nearest whole number of links for the side of a square field of the same area.

3. Reckoning the length of the Suez Canal to be 60 miles, and its surface equal to a square mile, what is its average width in feet?

4. How many pieces of wall-paper, each 12 yards of $9\frac{1}{4}$ inches width, must be bought, to cover the walls of a room 29 feet 7 inches long, 16 feet 8 inches wide, and $13\frac{1}{2}$ feet high, neglecting deductions for door, windows, etc.?

5. What is the area of a sector of a circle, the length of the arc being 8·156 inches, and the radius 1 foot?

PAPER 3.

1. Find, in perches, the area of a triangle whose sides are 115, 533, and 594 yards.

2. What is the area of a circle the circumference of which is 53 inches?

3. How many acres are contained in a five-sided field, *ABCDE*, the side *AB* being 1224 links, the diagonals *AD* 1184, *BD* 1400, and the perpendiculars on *AD* and *BD*, from *E* and *C* respectively, 256 and 320 links?

4. If 3 square feet be the area of a trapezoid whose parallel sides are 29 and 17 inches, what is the distance between the parallels?

5. The diameter of a circle is 1 foot. By how many inches does the perimeter of a square that is equal in area to the circle, exceed the circumference of the circle?

Paper 4.

1. Find the area of the quadrilateral *ABCD*, having given the perpendiculars on *AD* from *B* and *C*, viz., $BE=29$ feet, and $CF=26$ feet; also $AE=29$ feet, $EF=37$ feet, and $FD=8$ feet.

2. Two adjacent sides of an oblique parallelogram are 27 and 49 inches, and the angle between them is 45°. Find the area of the parallelogram.

3. How many square inches of gold leaf will gild a spherical surface of which the diameter is 18 inches?

4. The sides of a triangle are 36, 29, and 25 feet respectively. Determine the perpendicular distance of the longest side from the angular point opposite.

5. If the area of a square be 769 square inches, what will be the area of an equilateral triangle described on one of the sides of the square?

6. How many yards of brass wire, $\frac{1}{30}$ of an inch thick, will weigh half an ounce, reckoning the weight of a cubic foot of brass to be 8540 ounces?

Paper 5.

1. Find in square yards the area of a quadrilateral field *ABCD*, the sides *AB* and *BC* being respectively 53 and 35 yards, the diagonal *AC* 66 yards, and *DE*, perpendicular to the diagonal, 12 yards.

2. In a quadrilateral figure *ABCD*, the sides *AB* ($5\frac{1}{4}$ inches) and *DC* (7 inches) are both perpendicular to *AD*; what must be the length of *AD*, that the area of a figure may be equal to that of a square whose side is $17\frac{1}{2}$ inches?

3. Suppose a bar of metal, $13\frac{1}{2}$ inches long, $6\frac{1}{4}$ in. broad, and $1\frac{3}{4}$ in. thick, to be melted without loss into a cube; what difference would be made in the amount of surface?

4. Find, in cubic feet, the volume of a sphere of which the diameter is $14\frac{8}{9}$ inches.

5. The dimensions of a rectangle are 111 and 39 inches; find that an equilateral triangle, equal to it in area, is almost exactly equal to it in perimeter also.

6. A conical glass $4\frac{1}{2}$ inches deep is half-full of wine; how high does the liquid stand in the glass?

PAPER 6.

1. Define *trapezium, chord, arc, concentric rings, offset, prism, a regular figure, trapezoid.*
2. Find, by duodecimals, the area of a room, the length being 39 ft. 10 in., the breadth 19 ft. 4 in. 2 pt., and express the result in square feet and fractions of a square foot.
3. Find the area of an equilateral triangle whose sides are 1210 yards long, in acres, roods, etc.
4. A circular field has a diameter of 800 feet; how many rails, measuring $6\frac{1}{2}$ feet long, would be required to enclose it? Also, find its area in acres.
5. A cutting, 12 miles long, is of these dimensions:—top width 84 ft., bottom width 48 ft., depth 27 ft.; find the number of cubic yards of earth to be removed.
6. How many cubic yards are there in a cone whose base is 70 ft. in circumference, and the slant height is 75 ft.?
7. Plan and find the area of a field from the following particulars:—

	CHAINS.	
	to E	
	60	
to D 30	140	
	60	80 to C
to B 20	100	
	from A	

PAPER 7.

1. How many feet in height is an isosceles triangle, of which the base is 253 inches, and the area equal to that of another triangle whose sides are 184, 165, and 157 inches?
2. The area of a quadrilateral figure is $171\frac{1}{8}$ square inches, the perpendiculars on a diagonal, from the angles which it subtends, being 9 and $9\frac{1}{2}$ inches. What is the length of that diagonal?
3. A circular plate of iron weighs $4\frac{5}{8}$ ounces per cubic inch; it is $14\frac{1}{8}$ inches in diameter, and 2 inches thick. Find its whole weight in pounds.
4. Find the cost of painting the convex surfaces of five cylindrical pillars, each 14 feet high and a foot in diameter, at $8\frac{1}{2}$d. per square yard.
5. Find, to two places of decimals, the slant height of a cone, 19·25 cubic feet in volume, and 3·5 feet in the base diameter.
6. The area of an isosceles triangle is 98 square inches, each angle at the base being 45°. Find the length of the base.

7. The diameter of a sphere is 6 feet. How many cubic feet of it must be removed, that the remainder may form the largest cube that can be cut from it?

PAPER 8.

1. A rectangular room is 19 feet 6 inches long, and 15 feet 9 inches broad; how many yards of carpet, 27 inches wide, will be required to cover the floor?

2. Two sides of a triangle are at right angles, and measure 72 feet and 65 feet respectively; find the area of the triangle in square yards, and the length of the third side.

3. The lengths of the sides of a triangular field are 850, 970, and 1080 links respectively; how many acres, roods, and perches does it contain?

4. The diagonal AC of a quadrilateral $ABCD$ measures 320 yards, the perpendicular from angle B to the diagonal is 176 yards, and from angle D 224 yards; what is the area in acres?

5. A circular table measures 67·5 inches across; what is its circumference? And what is its surface in square feet?

6. How many square yards of canvas are required for a conical tent, the *vertical* height of which is 11·2 feet, and the diameter of the base, which is circular, 13·2 feet?

7. The diameter of a cast-iron ball is 8·4 inches; what is its weight in pounds, if the weight of a cubic foot of iron is 7000 ounces?

8. A circular bowling-green is to contain two acres, and is to be surrounded by a uniformly wide path, the area of which is to be half an acre; find the radius with which the circumference of the bowling-green may be described, and the width of the path.

PAPER 9.

1. A path 10 yards wide goes round a square whose area is 7225 square yards. Find how many stones 1 foot 4 inches long by 10 inches wide will be required to pave the path.

2. The expense of having a footpath 8 feet 3 inches wide on each side of a street, at the rate of 3s. 3d. a square yard, was £587 17s. 6d. Find the length of the street.

3. A triangular piece of ground, the base of which measures 136 feet, was sold at the rate of 4s. 6d. per square yard and realised £93 10s. What was the altitude of the triangle?

4. A piece of ground contained ·27104 of an acre and its length was 176 links. Find its breadth and its value at $1\frac{1}{2}$d. a square link.

5. One side of a right-angled triangle is 119 feet. Find the other side, if its area be the same as that of a triangle whose sides are 20, 493 and 507 feet.

6. The diagonal of a square measures 8 feet 4 inches. Find its area in square yards, feet and inches.

7. Find the area of a right-angled triangle of which the hypotenuse is 10 feet 5 inches and a side 9 feet 9 inches.

8. Find the area of a rectangle having diagonal 4 feet 5 inches long, and a side 2 feet 4 inches.

9. Find the area of an isosceles triangle of which each of the equal sides 7 feet 1 inch and the base 6 feet.

10. Find the height of a parallelogram having an area of $3\frac{1}{2}$ acres and a base of 242 yards.

11. A ladder 25 feet long stands upright against a wall. Find how far the bottom of the ladder must be pulled out from the wall so as to lower the top 1 foot.

12. Find the side of a square whose area is equal to the area of a rhombus whose diagonals are 52 feet and 416 feet.

Paper 10.

1. The length of a rectangular field is to its breadth as 6 to 5. One sixth of the field was planted, which left 625 square yards for ploughing. What is the length?

2. The extremity of the minute hand of a clock moves 5 inches in $3\frac{1}{4}$ minutes. What is its length?

3. $ABCD$ is a trapezium, of which the diagonal AC is 325 yards, and the sides AB, BC, CD, DA, are 123, 208, 116, 231 yards, respectively. Find the area in acres, roods and poles.

4. A rectangular field is 440 yards long and 154 yards wide; find its area in acres. Also find the areas of the portions into which it is divided by a straight line drawn from the middle point of one side to one of the opposite corners.

5. If a pressure of 15 lbs. on every square inch be applied to a circular plate 3 feet in diameter, find the total pressure to the nearest hundredweight.

6. Find the cost of boarding a floor in the form of a trapezoid whose parallel sides are 16 feet 8 inches and 14 feet 10 inches, and the distance between them 8 feet 4 inches, at $3\frac{1}{2}$d. per foot.

7. In the middle of a circular court, whose diameter is 112 feet, is a square whose side is 8 feet. Find the cost of paving the remainder of the court at 1s. 9d. per square yard.

8. There are two grass plots in a garden, one circular and the other in the form of a square; each has the same perimeter. Find the area of the circular plot, if the square contains 22 square yards.

9. Find the expense of planting the circumference of a circular piece of ground whose diameter is 126 feet with quicks one yard apart at 1s. 6d. each.

10. Find in acres the area of a circular ring whose inner diameter is 9 chains 45 links and outer diameter 12 chains 25 links.

11. The sides of a quadrilateral $ABCD$ are, in yards, $AB=16$, $BC=63$, $CD=33$, $DA=56$; the angle ABC is a right angle. Find the area in square yards.

12. Find the cost of making a road a rod wide and 300 chains long, if the land cost £140 an acre and the construction £4 4s. per square chain.

ANSWERS.

Exercise 1. (Page 5.)

1. 1623 links.
2. 2305 links.
3. 1025 links.
4. 8604 links.
5. 43 ch. 50 links.
6. 36 ch. 5 links.
7. 15·29 ch.
8. 27·5 ch.
9. 43·03 ch.
10. 616 yds.
11. 93·5 yds.
12. 121 yds.
13. 6 ch. 6 yards.
14. 610 ch. 20 yards.
15. 85¾ miles.
16. 8¾ miles.
17. 765 ch.
18. 415 ch.

Exercise 2. (Page 6.)

1. 8 ac. 3 ro. 26 per.
2. 246 ac. 2 ro. 32 per.
3. 5 ac. 2 ro. 27 per.
4. 7 ac. 3 ro. 38 per.
5. 4 ac. 1 ro. 18 per.
6. 765 ac. 3 ro. 9 per.

Exercise 3. (Page 6.)

1. 3 ch. 625 links.
2. 3 ac. 4 ch. 7500 links.
3. 7 ac. 5 ch. 2500 links.
4. 10 ac. [illegible] ch. 6875 links.
5. 12 ac. 8 ch. 6250 links.
6. 25 ac. 9 ch. 1250 links.

Exercise 4. (Page 7.)

1. [illegible]24 sq. ft.
2. 357 sq. ft.
3. 805 sq. ft.
4. 1512 sq. ft.
5. 2544 sq. ft.
6. 3380 sq. ft.
7. 4636 sq. ft.
8. 6586 sq. ft.
9. 7820 sq. ft.
10. 7280 sq. ft.
11. 10836 sq. ft.
12. 22365 sq. ft.

Exercise 5. (Page 7.)

1. 225 sq. ft.
2. 729 sq. ft.
3. 2025 sq. ft.
4. 2601 sq. ft.
5. 3969 sq. ft.
6. 7056 sq. ft.
7. 11025 sq. ft.
8. 56169 sq. ft.
9. 114244 sq. ft.

Exercise 6. (Page 8.)

1. 1 sq. ft. 51 sq. in.
2. 3 sq. ft. 62 sq. in.
3. 183 sq. ft.
4. 226 sq. ft. 48 sq. in.
5. 187 sq. ft. 72 sq. in.
6. 492 sq. ft.
7. 2530 sq. ft. 96 sq. in.
8. 2179 sq. ft. 120 sq. in.
9. 71 sq. ft. 1[illegible] sq. in.
10. 182 sq. ft. 66 sq. in.
11. 161 sq. ft. 96 sq. in.
12. 290 sq. ft. 40 sq. in.
13. 540 sq. ft. 54 sq. in.
14. 966 sq. ft. 132 sq. in.

Exercise 7. (Page 8.)

1. 262 sq. ft. 72 sq. in.
2. 108 sq. ft. 72 sq. in.
3. 30 sq. ft. 54 sq. in.
4. 10 sq. ft. 9 sq. in.
5. 69 sq. ft. 54 sq. in.
6. 43 sq. ft. 18 sq. in.

Exercise 8. (Page 9.)

1. 2 sq. ft. 1 sq. in.
2. 5 sq. ft. 121 sq. in.
3. 18 sq. ft. 112 sq. in.
4. 210 sq. ft. 86 sq. in.
5. 351 sq. ft. 81 sq. in.
6. 689 sq. ft. 9 sq. in.
7. 850 sq. ft. 100 sq. in.
8. 1045 sq. ft. 64 sq. in.
9. 1838 sq. ft. 49 sq. in.

Exercise 9. (Page 9.)

1. 20 sq. yds.
2. 91 sq. yds.
3. 15 sq. yds. 5 sq. ft.
4. 38 sq. yds. 2 sq. ft.
5. 390 sq. yds.
6. 76 sq. yds. 6 sq. ft.
7. 51 sq. yds.
8. 21 sq. yds. 8 sq. ft. 90 sq. in.
9. 19 sq. yds. 5 sq. ft.
10. 28 sq. yds. 4 sq. ft. 72 sq. in.
11. 52 sq. yds. 2 sq. ft. 86 sq. in.
12. 21 sq. yds. 8 sq. ft. 90 sq. in.

Exercise 10. (Page 9.)

1. 10 sq. yds. 86 sq. in.
2. 42 sq. yds. 9 sq. ft. 36 sq. in.
3. 68 sq. yds. 4 sq. ft. 100 sq. in.
4. 14 sq. yds. 6 sq. ft. 86 sq. in.
5. 282 sq. yds. 5 sq. ft. 9 sq. in.
6. 955 sq. yds. 7 sq. ft. 81 sq. in.

Exercise 11. (Page 9.)

1. 14 sq. ft.
2. 18 sq. ft. 112 sq. in.
3. 858 sq. ft. 54 sq. in
4. 22 sq. yds.
5. 2 sq. yds. 2 sq. ft.
6. 190 sq. ft. 129 sq. in.
7. 497 sq. yds.
8. 219 sq. yds. 8 sq. ft. 18 sq. in.
9. 9 sq. ft. 54 sq. in.
10. 5622 sq. ft. 108 sq. in.
11. 7 sq. ft. 45 sq. in.

Exercise 12. (Page 10.)

1. 14 ac.
2. 5 ac. 1 ro. 32 per. 22 yds.
3. 1 ac. 1 ro. 10 per.
4. 7 ac. 3 ro. 20 per.
5. 3 ac. 2 ro. 20 per.
6. 4 ac. 21 per. 4 yds. 6 ft. 108 in.

Exercise 13. (Page 10.)

1. 40 ac.
2. 2 ac. 2 ro.
3. 8 ac. 16 per.
4. 2 ac. 4 per.
5. 1 ac. 2 ro. 33 per. 22 yds. 6 ft. 108 in.
6. 18 ac. 2 ro. 17 per. 23 yds 4 ft. 108 in.

Exercise 14. (Page 10.)

1. 2 ac. 16 per.
2. 4 ac. 1 ro. 22 per.
3. 9 ac. 3. ro. 17½ per.
4. 30 ac. 3 ro. 16 per.
5. 4 ac. 2 ro.
6. 16 ac. 2 ro. 10 per.
7. 18 ac. 34 per.
8. 4 ac. 1. ro. 39 per.
9. 3 ac. 1 ro. 20 per.
10. 28 ac. 1 ro. 30 per.

Exercise 15. (Page 11.)

1. 7 ac. 36 per.
2. 5 ac. 2 ro. 20 per.
3. 105 ac. 2 ro. 20 per.
4. 11 ac. 4 per.
5. 56 ac. 1 ro. 25 per.
6. 124 ac. 1 ro. 1 per.

Exercise 16. (Page 13.)

1. 3 ft. 6′.
2. 7 ft. 5′ 8″.
3. 40 ft. 9′ 0″.
4. 15 sq. ft. 2′ 1″.
5. 46 sq. ft. 4′ 5″.
6. 110 sq. ft. 9′ 8″.

Exercise 17. (Page 13.)

1. 6 ft. 7 in.
2. 25 ft. $8\frac{1}{3}$ in.
3. 47 ft. $2\frac{2}{3}$ in.
4. 4 sq. ft. 41 sq. in.
5. 11 sq. ft. $108\frac{1}{4}$ sq. in.
6. 88 sq. ft. $80\frac{1}{4}$ sq. in.

Exercise 18. (Page 13.)

1. 17 sq. ft. 98 sq. in.
2. 72 sq. ft. 72 sq. in.
3. 261 sq. ft. 86 sq. in.
4. 251 sq. ft. 81 sq. in.
5. 598 sq. ft. 128 sq. in.
6. 380 sq. ft. 18[illegible] sq. in.
7. 75 sq. ft. 135 sq. in.
8. 416 sq. ft. 20 sq. in.
9. 72 sq. ft. 134 sq. in.
10. 202 sq. ft. $102\frac{1}{4}$ sq. in.
11. 100 sq. ft. $108\frac{3}{4}$ sq. in.
12. 848 sq. ft. $2\frac{1}{4}$ sq. in.

Exercise 19. (Page 14.)

1. 1848 ft.
2. 36 ft. 9 in.
3. 10 yds.
4. [illegible]7 yds.
5. 1250 links.
6. 94 ch. 50 links.

Exercise 20. (Page 14.)

1. 1 yd.
2. 15 ft. 9 in.
3. 11 yds. 1 ft.
4. 70 yds.
5. 625 links.
6. 26 ch. 50 links.

Exercise 21. (Page 14.)

1. 1 ft. $9\frac{1}{2}$ in.
2. 88 ft.
3. $48\frac{2}{3}$ ft.
4. 147·15 ft.
5. $17\frac{1}{2}$ ft.
6. $1\frac{1}{2}$ yds.

Exercise 22. (Page 15.)

1. 23 ft.
2. 98 ft.
3. 258 ft.
4. 216 yds.
5. 908 yds.
6. 74[illegible]
7. 11·9 ft.
8. 4·44 ft.
9. 48·32 ft.
10. $8\frac{1}{4}$ yds.
11. $1\frac{1}{3}$ yds.
12. $10\frac{1}{3}$ yds.

Exercise 23. (Page 15.)

1. 1 ft. 11 in.
2. 2 ft. 5 in.
3. 3 ft. 9 in.
4. 12 ft. 3 in.
5. 24 ft. 6 in.
6. 85 ft. 10 in.
7. 6 yds. 1 ft. 7 in.
8. 5 yds. 0 ft. 3 in.
9. 36 poles.
10. 352 yds.
11. 127 yds.
12. 729 yds.

Exercise 24. (Page 16.)

1. 770 yds.
2. 316·27 links.
3. 40 poles.
4. 71 yards.
5. 34 ch.
6. 65 ft. 9 in.

Exercise 25. (Page 16.)

1. 176 yds.
2. 124 yds.
3. 2112.
4. 1 mile 22 yds.
5. 5 minutes 54 seconds.
6. 30 ch.

Exercise 26. (Page 17.)

1. 34 ft. and 68 ft.
2. 5 yds. 1 ft. and 10 yds. 2 ft.
3. 13 ft. 11 in. and 41 ft. [illegible] in.
4. 616 yds. and 308 yds.
5. 176 yds. and 616 yds.
6. 96 ft. and 144 ft.

Exercise 27. (Page 18.)

1. 756 sq. ft.
2. 564 sq. ft.
3. 50064 sq. yds.
4. 83600 sq. yds.
5. 8 ac. 72 yds.
6. 8 ac. 3 ro. 34 per. 25¼ yds.
7. 2 ac. 4 per.
8. 38100 sq. yds.

Exercise 28. (Page 18.)

1. £71 12s. 9d.
2. £567 2s. 3d.
3. £1905 15s.
4. £582 10s. 2¼d.
5. £996 13s. 7¼d.
6. £5 4s. 0 3/16d.
7. £17 18s. 9d.
8. £17 11s. 1½d
9. £10.
10. £628 1s.
11. £115 10s.
12. £18 15s.

Exercise 29. (Page 19.)

1. £83 16s. 10½d.
2. £25 7s. 2d.
3. £75 7s. 4d.
4. £10 6s. 3d.
5. £5 7s. 11d.
6. £3 5s. 0⅛d.
7. £14 10s. 3⅛d.
8. £117 15s.

Exercise 30. (Page 20.)

1. 720.
2. 116160.
3. 1080.
4. 5920.
5. 20.
6. 2000.
7. 76½.
8. 760320
9. £217 16s.
10. 480; £2 14s. 5¾d.
11. 19; £9 10s.
    − £7 2s. 6d.
    = £2 7s. 6d.
12. £144.

Exercise 31. (Page 21.)

1. 4½ yds.
2. 187 ft.
3. 15 yds 2 ft.
4. 15 ft 4 in.
5. £33.
6. £104 10s.

Exercise 32. (Page 21.)

1. 70 yds.
2. 25 yds.
3. 149 yds. 8 in.
4. £12 12s. 2¼d.
5. £4 13s. 6d.
6. £15 3s. 1½d.
7. 2 ft. 3 in.
8. 5¼ yds.
9. 13¾ yds.
10. 25 ft.
11. 15 ft. 9 in.
12. 4s. 9d.

Exercise 33. (Page 22.)

1. 14 sq. yds. 6 sq. ft.
2. 8s. 2¾d.
3. £1 1s. 11½d.
4. 24 sq. yds. 8 sq. ft.
5. 21¼ yds.
6. £80 16s. 6d.
7. £8 11s.
8. £4 1s. [£11 5s. −(£4 10s +£2 14s.)]

Exercise 34. (Page 24.)

1. 95 sq. yds. 8 sq. ft.
2. 57 sq. yds. 7 sq. ft.
3. £38 17s. 9½d.
4. £4 14s. 9½d.
5. £7 4s. 5¾d.
6. £3 14s.
7. £12 7s. 4d.
8. £15 1s. 2d.
9. £5 3s. 9d.
10. £120 7s. 6d.

Exercise 35. (Page 25.)

1. 8 pieces.
2. 45 pieces.
3. 112 yds.
4. 182 yds.
5. 28 pieces.
6. £5 5s. 5d.
7. £3 5s. 4d.
8. £1 2s. 10½d.
9. £4 5s. 3d.
10. £2 8s. 8d.
11. £9 3s. 9d.
12. £3 8s. 10d.

Exercise 36. (Page 26.)

1. 16 panes.
2. £1 3s. 8d.
3. £325 10s.
4. £47 16s. 8d.
5. £21 1s. 8d.
6. £168 12s.

Exercise 37. (Page 27.)

1. 222$\frac{7}{9}$ sq. yds.
2. 17 sq. ft. 36 sq. in.
3. 748 sq. ft. 18 sq. in.
4. 8$\frac{1}{2}$ ac.
5. 5$\frac{1}{4}$ ac.
6. 52 ac. 3 ro. [illegible]52 sq. yds.
7. 11 ac. 3 ro. 726 sq. yds.
8. £5 5s.

Exercise 38. (Page 28.)

1. 6 ft. 8 in.
2. 10 yds. 1[illegible]
3. 7 yds. 1 ft.
4. 12 ft. 6 in.
5. 246 yds.
6. 32 [illegible]h.

Exercise 39. (Page 30.)

1. 420 sq. ft.
2. 49368 sq. yds.
3. 2 sq. ft.
4. 10 sq. yds. 3 sq. ft. 138 sq. in.
5. 1 sq. ft. 24 sq. in.
6. 33 sq. yds. 3 sq. ft.
7. 1 ac. 2 ro.
8. 1 ac. 3 ro. 2 per.
9. 1 ac. 2 ro. 20 per.
10. 3 ac. 1 ro. 4 per.
11. 64·86 yards.
12. 32 sq. ft. 117 sq. in

Exercise 40. (Page 30.)

1. 40 sq. in.
2. 189 ac.
3. 2 sq. ft. 64$\frac{1}{2}$ sq. in.
4. £23 15s.
5. £5 11s. 6$\frac{3}{4}$d.
6. £26 10s. 8d.

Exercise 41. (Page 31.)

1. 56 ft.
2. 6 ft. 4 in.
3. 38 poles.
4. 25 ch.
5. 1 ft. 2 in.
6. 308 yds.

Exercise 42. (Page 32.)

1. 504 sq. ft.
2. 756 sq. ft.
3. 660 sq. ft.
4. 210 sq. yds
5. 990 sq. yds.
6. 3570 sq. yds.
7. 15 ac.
8. 69 ac.
9. 3 ac. 1 ro. [illegible]1 per.
10. 25 ac. 32 per.
11. 1 ac. 21 per.
12. 9 ac. 24 per.

Exercise 43. (Page 32.)

1. 126 sq. in.
2. 2$\frac{1}{4}$ ac.
3. £2 15s.
4. £55 11s. 6d.
5. £1 10s. 11$\frac{9}{10}$d.
6. £6 18s. 10$\frac{11}{25}$d.
7. 600 tiles.
8. 10 ac. 1 ro. 33 per.

Exercise 44. (Page 34.)

1. 20 in.
2. 45 ft.
3. 97 ft.
4. 595 ft.
5. 105 chains.
6. 1455 yards.
7. 4 ft. 4 in.
8. 2 ft. 11 in.
9. 4 ft. 7 in.
10. 17 yds. 1 ft.
11. 3 yds. 1 ft. 3 in.
12. 67·7 ch.

Exercise 45. (Page 34.)

1. 12$\frac{1}{2}$ ft.
2. 30 ft.
3. 45 ft.
4. 50 ft.
5. 24 ft.
6. 75 miles.
7. ·602 miles.
8. 50 miles.
9. 62·48 miles.
10. $\frac{4}{5}$ mile.

Exercise 46. (Page 35.)

1. 60 ft.
2. 76 ft.
3. 56 ft.
4. 72 ft.
5. 68 ch.
6. 800 links.
7. 4 ft.
8. 2 ft. 6 in.
9. 1 ft. 8 in.
10. 10$\frac{1}{2}$ yds.
11. 8·9 ft.
12. 2 ft. 8 in.

Exercise 47. (Page 36.)

1. 27 ft.
2. 3$\frac{1}{2}$ ft.
3. 36 ft.
4. 2 ft.
5. 40 ft.
6. 45 yds.
7. 56 ft.
8. 12 ft.

Exercise 48. (Page 36.)

1. 88400 sq. ft.
2. 21 sq. ft.
3. 2 sq. ft. 42 sq. in.
4. 2 ro. 8 per.
5. 1 ac. 2 ro. 36 per.
6. 4 ac. 1640 yds.

Exercise 49. (Page 37.)

1. 48 sq. in.
2. 3468 sq. ft.
3. 768 sq. yards.
4. 3 sq. ft. 108 sq. in.
5. 19 ft. 36 sq. in.
6. 78 sq. ft. 108 sq. in.

Exercise 50. (Page 38.)

1. 6050 sq. yds.
2. 618272 sq. yds.
3. 5 sq. yds. 2 sq. ft. 76 sq. in.
4. 5 ac.
5. 12$\frac{3}{4}$ ac.
6. 4$\frac{1}{2}$ ac.

Exercise 51. (Page 38.)

1. 4800 yds.
2. 8$\frac{2}{3}$ sq. ft.
3. 8 ac. 1908 yds.
4. £2 10s.
5. £5 15s.
6. 98 yds.
7. 178 yds.
8. $\frac{3}{4}$ mile.

Exercise 52. (Page 39.)

1. 750 sq. ft.
2. 52204 sq. ft.
3. 4800 sq. yds.
4. 1 ac.
5. 2 ac. 36 per. 11 yds.
6. 22 ac. 1 ro. 4 per.

Exercise 53. (Page 39.)

1. 5 ac. 2 ro. 17·768 per.
2. 104 feet.
3. 18824.
4. 36 ft.
5. 58 ch.
6. 800 links.

Exercise 54. (Page 40.)

1. 270$\frac{5}{8}$ sq. ft.
2. 8·66 ft.
3. 26·4 yds.
4. 28$\frac{4}{5}$ ft.
5. 625$\frac{1}{2}$ sq. ft.
6. 1428 in.
7. 76·68 yds.
8. Triangle, 504 sq. ft.; square, 1296 sq. ft.; difference, 792 sq. ft.
9. 3·85 ft.
10. 124 sq. yds. 6 sq. ft. 86 sq. in.
11. 60 ft.
12. 84494 sq. yds.

Exercise 55. (Page 42.)

1. 340 sq. yds.
2. 11 sq. yds. 8 sq. ft. 140 sq. in.
3. 22 sq. yds. 4 sq. ft. 52 sq. in.
4. 27 ac. 20 per. 11 yds.
5. 62 ac. 3 ro.
6. 2 ro. 14 per.

Exercise 56. (Page 42.)

1. 152075 sq. ch.
2. $6\frac{3}{4}$ ac.
3. $8\frac{3}{4}$ ac.
4. $17\frac{1}{2}$ in.
5. £207 14s.
6. £1 18s. $8\frac{1}{2}$d.

Exercise 57. (Page 43.)

1. 80 sq. yds.
2. 2 ac. 2 ro. 20 per.
3. 5 ac. 1 ro. 24 per.
4. 1800 sq. yds.
5. 2 ac. 27 per.
6. 1253 sq. yds.
7. 1253 sq. yds.
8. £31 4s.
9. £121.
10. £32 6s. $3\frac{3}{4}$d.

Exercise 58. (Page 45.)

1. 15 sq. ft. 90 sq. in.
2. 1102 sq. ft. 72 sq. in.
3. 14 ac. 3 ro. 25 per.
4. 268 ac. 1 ro. 20 per.
5. 15 ac. 1 ro. 32 per.
6. 5 ac. 2 ro. 35 per.

Exercise 59. (Page 47.)

1. ·2854 ac.
2. ·851 ac.
3. ·2257 ac.
4. ·85218 ac.
5. 1 ac. 1 ro. 20 per.
6. 3 ac. 2 ro. 28·8 per.
7. 1 ac. 1 ro. 14·4 per.
8. 2 ac. 3 ro. 12 per.
9. 3 ac. 1 ro. 4 per.
10. 1 ac. 1 ro. 18 per.

Exercise 60. (Page 48.)

1. 1728.
2. £31 8s. 11d. (nearly).
3. 148200.
4. £12 19s. 4d.
5. 11 ac. 2 ro. 37·79 per.
6. 20 ft.
7. 233 (approx.).
8. £207 14s.
9. 50 ft.
10. 220 yds; 55 yds.
11. 27 yds. (nearly).
12. 126.
13. 69 yds. 1 ft. 8·5 in.
14. 52·49 miles.
15. £28 4s.
16. 11[illegible].
17. $60\frac{1}{2}$ yds.
18. £2 8s. 9d.
19. £10 0s. 6d.
20. 7[illegible]68.
21. 7·154 sq. ft.
22. 216.
23. 3·741 in.
24. 463·7408 sq. ft.

Exercise 61. (Page 51.)

1. 198 ft.
2. 132 yds.
3. 308 yds.
4. 19[illegible] yds.
5. $59\frac{3}{8}$ ft.
6. $85\frac{1}{3}$ ft.
7. 11 ft. [illegible] in.
8. 11 yds. 1 ft. 10 in.
9. 28 yds. 2 ft. 6 in.

Exercise 62. (Page 51.)

1. 33 ft.
2. 23 ft. $6\frac{3}{4}$ in.
3. 40 in. (nearly).
4. 17 ft. 8 in.
5. $569\frac{1}{2}$ yds.
6. 24856 miles.
7. 1760.
8. 76 yds. 2 ft.
9. 1000.
10. 25210.
11. 360.
12. 2 ft. $7\frac{3}{4}$ in.
13. 88.
14. £168 12s. 6d.
15. £9 16s.
16. $9\frac{3}{4}$ miles.
17. 10.

Exercise 63. (Page 52.)

1. 77 ft.
2. 140 yds.
3. 12·25 ft.
4. 2[illegible] in.
5. 4 ft. 1 in.
6. 1 ch. 5 links.
7. $2\frac{5}{12}$ ft.
8. $25\frac{1}{3}$ yds.
9. 35 yds.
10. $8\frac{3}{16}$ ft.
11. 2 ft.
12. 4 ft. $4\frac{8}{11}$ in.

Exercise 64. (Page 52.)

1. 7·48 in.
2. 2180 miles.
3. 15 ch. 90[illegible] links.
4. $2\frac{1}{4}$ ft.
5. 3 ch. 85 links.
6. 91 yds.

Exercise 65. (Page 53.)

1. 1386 sq. ft.
2. 10028$\frac{2}{3}$ sq. ft.
3. 129 sq. yds. 8 ft. 90 in.
4. 65 ac. 2 ro. 1$\frac{3}{5}$ per.
5. 38$\frac{1}{2}$ sq. ft.
6. £776 8s. 4d.

Exercise 66. (Page 54.)

1. 1386 sq. ft.
2. 21374 sq. ft.
3. 8850 sq. ft.
4. 154 sq. yds.
5. 129 sq. yds. 8 ft. 90 in.
6. 11 sq. yds. 7 ft. 136 in.
7. 707$\frac{1}{4}$ sq. ft.
8. 9 sq. ft. 90 in.
9. 26 sq. ft. 106 in.
10. 3118$\frac{1}{3}$ sq. ft.
11. 103$\frac{2}{5}$ sq. ft.
12. 346$\frac{1}{2}$ sq. ft.

Exercise 67. (Page 54.)

1. 6$\frac{27}{8}$ ac.
2. 2 ac. 2 ro. 12 per. 11 sq. yds.
3. 65 ac. 2 ro. 1$\frac{3}{5}$ per.
4. 14·186 sq. ft.
5. 7·071 sq. ch.
6. 314 sq. yds. 2$\frac{1}{4}$ ft.
7. £133 12s. 6d.
8. 9s. 0$\frac{1}{28}$d.
9. £5 5s. 9$\frac{11}{12}$d.
10. 436 tons.
11. 1 ac. 1 ro. 15·4 per.
12. £95 4s.

Exercise 68. (Page 55.)

1. 9$\frac{3}{5}$ sq. [illegible]
2. 2464 sq. ft.
3. 88 sq. ft. 142 in.
4. [illegible]0028$\frac{2}{3}$ sq. yds.
5. 273 sq. yds. 7 ft.
6. 129 sq. yds. 8 ft. 90 in.
7. 14 sq. yds. 8 ft. 58 in.
8. 84 sq. yds 3 ft. 10 in.
9. 3 sq. [illegible] 9424 links.
10. 15 ac. 1 ro. 24 per.
11. 29 ac. 20 per.
12. 2 ro. 27 3 per.

Exercise 69. (Page 56.)

1. 4402·12 sq. yds.
2. 28·27 sq. ft.
3. £2 9s. 9$\frac{3}{4}$d.
4. 838 sq. yds. 4 sq. ft.
5. £10 8s. 7$\frac{1}{4}$d.
6. 19 ac. 3 ro. 21 per. 24$\frac{3}{4}$ yds.

Exercise 70. (Page 56.)

1. 2 ft. 4 in.
2. 112 ft.
3. 1 ft. 2 in.
4. 4 yds. 2 ft.
5. 18 yds. 2 ft.
6. 22 ft. 2 in.
7. 28 ft.
8. 35 yds.
9. 9 ft. 4 in.
10. 5 ft.
11. 11 ft.
12. [illegible] 22 links.

Exercise 71. (Page 57.)

1. (1) 8 ch. 56 links; (2) 78 yds. 1 ft.
2. 1750 links.
3. 35 yds.
4. 11·28 yds.
5. 285·457 yds.
6. 49 yds.

Exercise 72. (Page 57.)

1. 132. ft.
2. 66 ft.
3. 29 yds. 1 ft.
4. 58 yds. 2 ft.
5. 16 ft. 6 in.
6. 33 ft.

Exercise 73. (Page 58.)

1. 88 ch.
2. 968 yds.
3. 6[illegible] min.
4. 44 [illegible]ls.
5. £18 15s.
6. 192.

Exercise 74. (Page 59.)

1. 2772 sq. ft.
2. 199 sq. yds. 7 sq. ft.
3. $1732\frac{1}{4}$ sq. ft.
4. 8 ac. 2 ro. 26 per.
5. $754\frac{3}{4}$ sq. ft.
6. $5854\frac{6}{11}$ sq. ft.
7. 892 sq. ft.
8. $15\frac{5}{8}$ sq. yds.
9. 2 ro. 18 per. 14 yds.
10. £1 18s. $9\frac{1}{4}$d.
11. £47 2s. $10\frac{3}{4}$d.
12. 407·01173.

Exercise 75. (Page 60.)

1. 12 in.
2. $4\frac{1}{8}$ in.
3. 22 yds.
4. 11 in.
5. 8 ft.
6. 6 ft. 5 in.
7. 5 in.
8. $8\frac{1}{2}$ ft.
9. 6 ft. 5 in.

Exercise 76. (Page 61.)

1. 45°.
2. 15°.
3. 36°.
4. 72°.
5. 48°.
6. 78° 45′.
7. 11° 15′.
8. 57° 17′ 44″.

Exercise 77. (Page 62.)

1. 189 sq. yds. 6 ft. 96 in.
2. 385 sq. ft.
3. 34 sq. yds. 2 sq. ft.
4. $58\frac{1}{3}$ sq. ft.
5. 24 sq. ft. $36\frac{4}{25}$ sq. in.
6. 78 sq. yds. 5 sq. ft. 63 sq. in.
7. 95 sq. yds. 5 sq. ft. 54 sq. in.
8. $85\frac{1}{2}$ ft.
9. 63 ft.
10. $24\frac{1}{2}$ ft.
11. 30°.
12. 80°.

Exercise 78. (Page 63.)

1. 90 sq. ft.
2. 14 sq. ft.
3. 882 sq. ft.
4. 85 sq. yds. 8 ft. 108 in.
5. 18 ft.
6. 8 ft. 6 in.

Exercise 79. (Page 64.)

1. £3 1s. $10\frac{1}{2}$d.
2. £424 9s. 3d.
3. $19\frac{1}{11}$ ft.
4. 3 ac. 1 ro. 902 yds.
5. 21 ft.
6. 240.
7. 30184 sq. yds.
8. 189.
9. $14\frac{1}{4}$ (nearly).
10. $95\frac{5}{11}$.

Exercise 80. (Page 66.)

1. 2197 cub. yds.
2. 9261 cub. yds.
3. 4·096 cub. yds.
4. 91·125 cub. yds.
5. 12 cub. yds. 19 cub. ft.
6. 41 cub. yds. 23 cub. ft. 485 cub. in.
7. 18 cub. yds. 26 cub. ft.
8. 81 cub. yds. 10 cub. ft.
9. 20 cub. ft. 1377 cub in.
10. 2 cub. yds. 2 cub. ft. 568 cub. in.
11. 6 cub. yds. 4 cub. ft. 648 cub. in.
12. 10 cub. yds. 26 cub. ft. 512 cub. in.

Exercise 81. (Page 66.)

1. 4096 cub. ft.
2. 9261 cub. ft.
3. 110592 cub. in.
4. 11 cub. ft. 675 cub. in.
5. 18s. $0\frac{1}{4}$d.
6. 72 cub. yds. 9 cub. ft. 216 cub. in.

Exercise 82. (Page 67.)

1. 18 in.
2. 24 in.
3. 32 in.
4. 48 in.
5. 2·6 ft.
6. 7·9 ft.
7. $3\frac{3}{4}$ ft.
8. $5\frac{5}{8}$ yds.
9. 2 ft. 6 in.
10. 3 ft. 6 in.
11. 2 ft. 3 in.
12. 3 ft. 4 in.

Exercise 83. (Page 67.)

1. 25 ft.
2. 27 in.
3. 5 ft. 9 in.
4. 4 ft. 3 in.
5. 8 in.
6. 15 ft. 4 in.

Exercise 84. (Page 68.)

1. 2400 sq. ft.
2. 2 sq. ft. 96 sq. in.
3. 66 sq. ft. 8 sq. in.
4. 9 sq. ft. 54 sq. in.
5. 104 sq. ft. 24 sq. in.
6. 74 sq. yds. 6 sq. ft. 6 sq. in.

Exercise 85. (Page 68.)

1. £1 2s. 6d.
2. £1 8s. 7d.
3. £3 18s. 6d.
4. $66\frac{2}{3}$ sq. ft.
5. 9d.
6. £4 5s. $6\frac{3}{4}$d.
7. £2 12s.
8. 4 ft. 2 in.
9. 42 cub. ft. [illegible] cub. in.
10. 46 cub. ft. [illegible] cub. in.

Exercise 86. (Page 69.)

1. 891 cub. ft.
2. 167 cub. ft. 864 cub. in.
3. 131 cub. ft. 9 cub. in.
4. 80 cub. ft. 1680 cub. in.
5. 6 cub. ft. 192 cub. in.
6. 229 cub. ft. 1153 cub. in.

Exercise 87. (Page 70.)

1. $42\frac{6}{7}$ cub. in.
2. 192[illegible] cub. ft.
3. 480 cub. yds.
4. 6650 cub. ft.
5. $8\frac{1}{4}$ cub. yds.
6. $50\frac{5}{12}$ cub. ft.
7. $\frac{29}{27}$ cub. ft.
8. 5 cub. ft.
9. £1 1s. $8\frac{1}{4}$d.
10. £2 16[illegible].
11. £5 2s. $4\frac{1}{2}$d.
12. £36 10s.

Exercise 88. (Page 70.)

1. 195[illegible].
2. 360.
3. 122392.
4. 1584.
5. £10 10s.
6. 48.

Exercise 89. (Page 72.)

1. 6 tons 2 qrs. 4 lbs.
2. 18 cwt. 15 lbs. 4 oz.
3. 25 tons.
4. $187\frac{1}{2}$.
5. 2917.
6. $4\frac{1}{2}$ tons.
7. 6767 tons.
8. 0 tons 8 cwt. 1 qr. 9 lbs. [illegible] oz.

Exercise 90. (Page 72.)

1. 6 cub. ft. 1243 cub. in.
2. 3 cub. ft. 816 cub. in.
3. 2 cub. ft. 1552 cub. in.
4. 7 cub. ft. 81 cub in.
5. 2835 cub. in.
6. 3780 cub. ft.

Exercise 91. (Page 73.)

1. 117 cub. ft. 1504 cub. in.
2. 2809 cub. ft. 198 cub. in.
3. 3330 cub. ft. 1620 cub. in.
4. 246 cub. ft. 162 cub. in.
5. 218 cub. yds. 9 cub. ft. 486 cub. in.
6. 850 cub. ft. 1480 cub. in.
7. 939 cub. ft. $1255\frac{1}{2}$ cub. in.
8. 329 cub. yds. 21 cub. ft. 278 cub. in.

Exercise 92. (Page 74.)

1. 9 ft.
2. 5·029 ft.
3. 5 in.
4. 2 ft. 3 in.
5. 4 yds. 2 ft.
6. 2 ft. 5 in.

Exercise 93. (Page 74.)

1. 8 yds.
2. $4\frac{1}{2}$ in.
3. 18 ft. 4 in.
5. 2[illegible]0 sq. ft.
6. [illegible] ft.
7. [illegible] ft. [illegible] in.
9. $28\frac{1}{2}$ sq. yds.
10. 24 ft.
11. 6 in.; 187 cub. ft.

Exercise 94. (Page 75.)

1. 6 sq. ft. 20 sq. in.
2. 8 sq. ft. 48 sq. in.
3. 152 sq. ft. 72 sq. in.
4. 520 sq. ft. 72 sq. in.
5. 127 sq. ft. 72 sq. in.
6. 82 sq. ft. 112 sq. in.

Exercise 95. (Page 76.)

1. £4 2s. 10½d.
2. £3 5s. 10¾d.
3. 8s. 8d.
4. £2 14s.
5. £37 16s. 4½d.
6. 10s. 2¾d.

Exercise 96. (Page 78.)

1. 99 cub. ft. 1152 cub. in.
2. 64·062 cub. ft.
3. 4200 cub. in.
4. 336 cub. ft.
5. 96 cub. ft. 482 cub. in.
6. 96 cub. in.
7. 13 cub. ft. 1476 cub in.
8. 8 cub. ft. 216 cub. in.

Exercise 97. (Page 78.)

1. 4.
2. £15 15s.
3. 252 cub. ft. 1176 cub. in.
4. 117333328¼ cub. yds.
5. 4 ft.
6. 8 sq. ft. 68 sq. in.

Exercise 98. (Page 79.)

1. 10 sq. ft. 48 sq. in.
2. 42 sq. ft. 22 sq. in.
3. 184 sq. in.
4. 20½ sq. yds.
5. £1 13s. 9d.
6. £9 16s. 10½d.

Exercise 99. (Page 81.)

1. 154 cub. in.
2. 154 cub. ft.
3. 1540 cub. in.
4. 48 cub. ft. 216 cub. in.
5. 26 cub. ft. 1248 cub. in.
6. 5 cub. ft. 908 cub. in.
7. 16 cub. ft. 1458 cub. in.
8. 107 cub. ft. 1296 cub. in.
9. 19 cub. ft. 1664 cub. in.
10. 404 cub. ft. 432 cub. in.

Exercise 100. (Page 81.)

1. $7\frac{4}{9}$ cub. yds.
2. £8 11s. 10½d.
3. £9 1s. 6d.
4. 3 cub. ft. 860 cub. in.
5. 770 cub. ft.
6. 1540 cub. in.
7. 42 cub. ft. 1344 cub. in.
8. $488\frac{4}{11}$.
9. 2887 gallons.
10. £4 1s.

Exercise 101. (Page 82.)

1. $4\frac{121}{128}$ cub. ft.
2. 2 cub. ft. 702 cub. in.
3. 7 cub. ft. 609 cub. in.
4. 24 cub. ft. 108 cub. in.
5. 808 cub. in.
6. 18 cub. ft. 576 cub. in.

Exercise 102. (Page 83.)

1. 12 ft.
2. 3 sq. ft. 68 in.
3. 5 ft.
4. 10½ in.
5. 4 ft. 8 in.
6. 14 ft. 8 in.
7. 7392 yds.
8. 20 ft.
9. 9 in.
10. 2 ft. $9\frac{5}{7}$ in.

Exercise 103. (Page 84.)

1. 1 sq. yd. 8 sq. ft. 72 sq. in.
2. 12 sq. yds. 2 sq. ft.
3. 36 sq. ft. 96 sq. in.
4. 17 sq. yds. [illegible] sq. ft. 128 sq. in.
5. 21 sq. yds. 2 sq. ft. 106 sq. in.
6. 7 sq. yds. 80 sq. in.

Exercise 104. (Page 85.)

1. 17s. 3$\frac{3}{4}$d.
2. £18 6s. 8d.
3. £3 2s. 10$\frac{3}{4}$d.
4. £1 1s. 2$\frac{1}{4}$d.
5. 352 sq. ft.
6. 1540.
7. $\frac{1}{2}$ acre.
8. 1 acre.

Exercise 105. (Page 87.)

1. 7500 cub. ft.
2. 289 cub. ft.
3. 13 cub. ft. 405 cub. in.
4. 29 cub. ft. 1080 cub. in.
5. 12 cub. ft.
6. 108 cub. ft. 432 cub. in.
7. 1 cub. ft.
8. 145·488 cub. ft.

Exercise 106. (Page 87.)

1. 22 cub. ft. 792 cub. in.
2. 21 tons 17 cwt. 2 qrs.
3. £55 2s. 6d.
4. 3419010 cub. yds. 1[illegible] cub. ft. 1152 cub. in.
5. 26 cub. ft. 986 cub. in.
6. 1 ton 12 cwt. 16 lbs.

Exercise 107. (Page 89.)

1. 1080 sq. ft.
2. 8 sq. ft. 45 sq. in.
3. 744 sq. ft.
4. 26$\frac{1}{4}$ sq. ft.
5. 49 sq. ft. 84 sq. in.
6. 177·588 sq. ft.
7. 280 sq. ft. 10 sq. in.
8. 1040 [illegible]. ft.
9. £1[illegible] 13s. 4d.

Exercise 108. (Page 90.)

1. 1 cub. ft. 1352 cub. in.
2. 2 cub. ft. 8[illegible]3 cub in.
3. 24 cub. ft. 108 cub. in.
4. 2515 cub. ft. 576 cub. in.
5. 49 cub. ft. 28 cub. in.
6. 51 cub. ft. 576 cub. in.
7. [illegible]3 cub. ft. 72 cub. in.
8. 1232 cub. ft.
9. 59 cub. ft. 1536 cub. in.
10. 21 cub. ft. 672 cub. in.

Exercise 109. (Page 90.)

1. 770 cub. in.
2. 128 cub. ft. 576 cub. in.
3. 209$\frac{1}{2}$ cub. ft.
4. 3$\frac{1}{7}$ cub. in.
5. 103488 cub. yds.
6. 1880[illegible].

Exercise 110. (Page 92.)

1. 550 sq. ft.
2. 19 sq. ft. 36 sq. in.
3. 32 sq. ft. 56 sq. in.
4. 7 sq. ft. 48 sq. in.
5. 99 sq. ft.
6. 40 sq. ft. 136 sq. in.
7. 15 sq. ft. 40 sq. in.
8. 527$\frac{5}{11}$ sq. ft.
9. 6737$\frac{1}{2}$ sq. ft.
10. 188$\frac{4}{7}$ sq. ft.

Exercise 111. (Page 92.)

1. £17.
2. 3 cub. ft. 1116 cub. in.
3. 14$\frac{2}{9}$ sq. yds.
4. 48 sq. yds. 6 sq. ft. 134 sq. in.
5. £1 7s. 6d.
6. 326$\frac{2}{3}$.

Exercise 112. (Page 94.)

1. 179 cub. ft. 1152 cub. in.
2. 22 cub. ft. 792 cub. in.
3. 75 cub. ft. 1377 cub. in.
4. 12 cub. ft. 1722$\frac{1}{2}$ cub. in.
5. 16[illegible]7$\frac{1}{2}$ cub. in.
6. [illegible]6 cub. ft. 648 cub. in.
7. [illegible]07 cub. ft. 504 cub. in.
8. [illegible] cub. ft. 824 cub. in.
9. 9 cub. ft. 1696 cub. in.
10. 2 cub. ft. 1395 cub. in.
11. 35[illegible] cub. ft. 1575 cub. in.
12. 138 cub. ft. 672$\frac{1}{2}$ cub. in.

Exercise 113. (Page 94.)

1. $381\frac{4}{7}$ cub. in.
2. 18 lbs. $6\frac{8}{13}$ oz.
3. 1 cwt. 86 lbs.
4. $20\frac{28}{81}$ gall.
5. 114048 lbs.
6. 67 tons 18 cwt. 2 qrs. 1 lb. 12 oz.

Exercise 114. (Page 95.)

1. 9856 sq. ft.
2. 4 sq. ft. 40 sq. in.
3. 427 sq. ft. 112 sq. in.
4. 26 sq. ft. 106 sq. in.
5. 616 sq. ft.
6. 17 sq. ft. 16 sq. in.
7. 88 sq. ft. 72 sq. in.
8. 864 sq. ft. 72 sq. in.

Exercise 115. (Page 95.)

1. $1018\frac{2}{3}$ sq. in.
2. 6[illegible] sq. ft. $98\frac{1}{2}$ sq. in.
3. £29 14s.
4. £11 11s.
5. £237 12s.
6. 693 sq. ft.

Exercise 116. (Page 96.)

1. $1019\frac{1}{4}$ cub. in.
2. 1 cub. ft. 1704 cub. in.
3. $381\frac{4}{7}$ cub. in.
4. $709\frac{1}{2}$ lbs.
5. 66 lbs. $10\frac{1}{2}$ oz.
6. $163\frac{3}{4}$ lbs.

Exercise 117. (Page 96.)

1. 1353 cub. in.
2. $760\frac{8}{33}$ bullets.
3. $1832\frac{7}{9}$ ft.
4. $7\frac{13}{18}$ oz.
5. £26 14s.
6. $600\frac{1}{6}$ cub. ft.
7. 102 cub. yds. 21 cub. ft.
8. 1750.
9. 4 cwt. 3 qrs. $4\frac{1}{2}$ lbs.
10. 112640 cub. yds.
11. £45.
12. 23520.

Exercise 118. (Page 97.)

1. 9000.
2. £2 15s.
3. [illegible] cub. ft.
4. 15s. $1\frac{1}{2}$d.
5. 796 in.
6. 1440 cub. in.
7. 8 cub. ft. $656\frac{2}{3}$ cub. in.
8. 43552.
9. 667·6 sq. ft.
10. [illegible] 15s.
11. $1\frac{13}{22}$ cub. ft.
12. 5 cub. yds. 9 cub. ft. 648 cub. in.
13. 90.
14. $10\frac{1}{2}$ sq. ft.
15. 2673.
16. 788480; 2112.
17. 9 in.
18. 847 sq. ft.
19. £2 5s. $1\frac{7}{8}$d.
20. 49 sq. ft. 84 sq. in.
21. 10200.
22. 472 sq. ft.
23. $5\frac{1}{2}$ ft.
24. 1 cub. ft. 1272 cub. in.

Examination Paper 1. (Page 99.)

1. $2\frac{1}{4}$ ac.
2. 5·2355 sq. ft.
3. 16·837 in.
4. 292·7166 sq. in.

Examination Paper 2. (Page 99.)

1. 6·49519 sq. ft.
2. 845 links.
3. 88 ft.
4. 45.
5. 48·936 sq. in.

Examination Paper 3. (Page 99.)

1. $903\frac{1}{11}$ per.
2. 1 sq. ft. $79\frac{1}{2}$ sq. in.
3. 10·61 ac.
4. 18·73 in.
5. 4·84 in.

Examination Paper 4. (Page 100.)

1. 1542 sq. ft.
2. $935\frac{1}{2}$ sq. in.
3. 1017·876.
4. 20 ft.
5. 332·987 sq. in.
6. 3·22 yds.

Examination Paper 5. (Page 100.)

1. 1320 sq. yds.
2. 50 in.
3. $68\frac{1}{7}$ sq. in.
4. 1 cub. ft.
5. ——
6. 3·57165 in.

Examination Paper 6. (Page 101.)

1. See text.
2. $770\frac{21}{64}$ sq. ft.
3. 130 ac. 3 ro. 37·8 per.
4. 387 ; $11\frac{31}{64}$ ac.
5. 4181760 cub. yds.
6. 356 cub. yds. (approx
7. 2130 ac.

Examination Paper 7. (Page 101.)

1. 8 ft.
2. $18\frac{1}{2}$ in.
3. $86\frac{3829}{17920}$ lbs.
4. 17s. $3\frac{3}{5}$d.
5. 6·25 ft.
6. 19·798 in.
7. 36·786 cub. ft.

Examination Paper 8. (Page 102.)

1. $45\frac{1}{2}$ yds.
2. 260 sq. yds. ; 97 ft.
3. 3·9308 ac.
4. $13\frac{37}{121}$ ac.
5. $212\frac{4}{7}$ in. ; 24·86059 sq. ft.
6. 30 (nearly).
7. 76·604 lbs.
8. $55\frac{1}{2}$ yds. ; $6\frac{1}{2}$ yds.

Examination Paper 9. (Page 102.)

1. 30780.
2. 657·76 yds.
3. 55 ft.
4. 154 links ; £169 8s.
5. 60 ft.
6. 3 sq. yds. 7 sq. ft. 104 sq. in.
7. 17 sq. ft. 126 sq. in.
8. 8 sq. ft. 108 sq. in.
9. 19 sq. ft. 36 sq. in.
10. 70 yds.
11. 7 ft.
12. 104 ft.

Examination Paper 10. (Page 103.)

1. 30 yds.
2. 14 in.
3. 2 ac. 3 ro. 28 per. 8 yds.
4. 14 ac. ; $10\frac{1}{2}$ ac. ; $3\frac{1}{2}$ ac.
5. $136\frac{3}{8}$ cwt.
6. £1 18s. $8\frac{2}{3}$d.
7. £95 4s.
8. 28 sq. yds.
9. £9 18s.
10. 4·774 ac.
11. 1428 sq. yds.
12. £1865.

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